First Midterm Exam Samples

1. Consider the differential operator L_M defined by

$$L_M\, u = -\frac{d^2 u(x)}{dx^2}$$

for all functions $u \in C_M^2[0,l] = \{u \in C^2[0,l] \mid u(0) = u'(l) = 0\}$. What are the eigenvalues and eigenfunctions of this differential operator?

2. Solve by the method of Fourier series:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$$

$$u(x,0) = \begin{cases} 1 & 0.4 \le x \le 0.6 \\ 0 & \text{otherwise} \end{cases}$$

$$u(0,t) = u(1,t) = 0$$

3. Explain how to use the finite element method to approximate the solution to the BVP

$$-\frac{d}{dx}\left[(1+x^2)\frac{du}{dx}(x)\right] = f(x)$$

$$u(0) = u(1) = 0$$

4. Consider this boundary value problem:

$$-\frac{d}{dx}\left[(1+x^2)\frac{du}{dx}(x)\right] = f(x)$$

$$u'(0) = u'(1) = 0$$

Show that the differential operator has a non-trivial null space and that this in turn forces us to demand that $f(x)$ satisfy a special condition for the equation to have a solution. What is that special condition?