Experimental Realization of Nearly Steady-State Toroidal Electron Plasmas
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APS-DPP ’08, Invited Talk UI1 1
Acknowledgements

• Joan Marler (Post-doctoral Fellow 2005-2007)
  – Present address: University of Aarhus, Denmark
• Bao Ha (LU undergraduate ’07)
  – Present address: California Institute of Tech.
• Jason Smoniewski (LU undergraduate ’09)
  – Undergraduate poster session: JP6 10
• National Science Foundation / Dept. of Energy
  – Grant Nos. PHY-0317412, PHY-0812893
Take Home Points

• *Theoretical Predictions:* Electron plasmas can be confined in a purely toroidal magnetic field.
  – Stable, maximum energy state equilibria exist and rely on the poloidal $ExB$ rotation acting as an effective rotational transform [1,2].
  – Magnetic pumping transport limits ultimate confinement time [3].

• *Experimental Results:* A new experiment (Lawrence Non-neutral Torus II) has demonstrated long-lived (>1 s) toroidal electron plasmas that approach the predicted maximum lifetime [4].

Non-neutral Plasmas: *The Penning-Malmberg Trap*

- Non-neutral plasmas: *electron plasmas, ion plasmas, positron plasmas.*
- Experimental “wind-tunnel” tests of plasma and (2D-) neutral fluid theory.
- Applications/connections: *neutral antimatter production, frequency standards, quantum computation*

Dynamic equilibrium:
- azimuthal $E \times B$ flow due to space charge provides inward $J \times B$ force to balance outward electrostatic force.
Toroidal Electron Plasmas

• Interest in toroidal electron plasmas pre-dates much of the work in Penning-Malmberg traps.

• Contemporary/recent experiments that investigate non-neutral plasmas in toroidal geometry:
  – Columbia Non-neutral Torus, New York: stellerator field
  – Compact Helical System, Japan: stellerator field
  – Proto-RT, Japan: (levitated) dipole field
  – Smartex-C, India: pulsed purely toroidal field … partial torus
  – Lawrence Non-neutral Torus II, Wisconsin: DC purely toroidal field
Physics Issues for Toroidal Electron Plasmas

• Equilibrium & Stability

• Dynamics

• Limitations on confinement
Equilibrium & Stability

- Daugherty-Levy Eq. [1]: \( \nabla^2 V = \frac{ef(V)}{\varepsilon_o R^2} \)
- Poloidal ExB rotation acts as an effective rotational transform.
- No banana orbits.
- Criteria for closed orbits: \( e \phi_{\text{plasma}} > kT \)
- Maximum energy state is stable because kinetic energy is constrained by invariants [2].

Dynamics (& Diagnostics):  
*Diocotron Modes* ($k_{||} = 0$)

$m=1$ Mode

$m=2$ Mode

Theory for cylinder:

\[ f_1 = \frac{Q}{4\pi^2\varepsilon_0 L B b^2} \left( \frac{1}{1 - (A_1/b)^2} \right) \]

- Measure trapped charge.

Theory for cylinder:

\[ f_2 \approx \frac{ne}{4\pi\varepsilon_0 B} = f_{\text{ExB}} \]

- Measure density.

**Toroidal effects?**
Limits on Confinement: *Magnetic Pumping Transport*


Adiabatic invariants/constants of the motion:

Magnetic moment \[ \langle \mu \rangle B_o R_o \]

Angular momentum \[ \frac{1}{2} T^2 = \frac{1}{2} \langle m v^2 \rangle = \frac{\langle L_z^2 \rangle}{2mR^2} \]

For each fluid element:

\[ T_\perp = \langle \frac{1}{2} m v_\perp^2 \rangle = \frac{\langle \mu \rangle B_o R_o}{R} \]

\[ T_\perp R = \text{constant} \]

\[ T_\parallel = \frac{1}{2} T^2 = \frac{1}{2} \langle m v_\parallel^2 \rangle = \frac{\langle L_z^2 \rangle}{2mR^2} \]

\[ T_\parallel R^2 = \text{constant} \]

- \[ \tilde{T}_\parallel = 2 \tilde{T}_\perp \]
- Collisional equilibration leads to heating.
- Energy source: electrostatic (space-charge) potential energy.

\[ \rightarrow \text{Plasma expands} \ldots \text{TRANSPORT.} \]

*Scaling analysis:*

\[ \tau_{mp} \approx 0.02 R_o \left( \text{cm} \right)^2 \sqrt{T(\text{eV})} \]

Independent of \( B, n, a \)!!

\[ \tau_{mp} \approx 6 \text{ s} \quad \text{For } R_o = 17.4 \text{ cm, } T = 1 \text{ eV} \]
Lawrence Non-neutral Torus II

- Vacuum \( \sim 10^{-9} \) Torr
- Magnetic field \( \sim 700 \) G
- Field symmetry / boundary conditions
- Flexible wall diagnostics and control
- Fully toroidal… eventually

- Plasma major radius: 17.4 cm
- Plasma minor radius: \( \sim 1.3 \) cm
- Length: 82 cm (270 degrees)
  109 cm (360 degrees)
Internal Electrodes and Partial Toroidal Trapping
Observation of $m=1$ Diocotron Mode

$$f_1 = \frac{Q}{4\pi^2\varepsilon_0 Lb^2} \left(\frac{1}{B}\right) \approx 50 \text{ kHz}$$

$Q \approx 1.5 \text{ nC}$

$N \approx 10^{10}$ electrons

Marler and Stoneking, *PRL* 100, 155001 (2008)
Measuring Confinement Time

- \( m=1 \) mode frequency \( \rightarrow \) charge
  \[
  f_1 = \frac{Q}{4\pi^2 \varepsilon_0 L b^2} \left( \frac{1}{B} \right)
  \]

- Launch (C5) with a 5 cycle, near-resonant tone burst.
- Mode damps on \( \sim 300 \text{ ms} \) timescale.
- Frequency is measured (C2) after the tone burst ceases.
Confinement Time

- Frequency decays on ~3 s timescale $\rightarrow$ charge confinement time.
- ~100X improvement over previous experiments.
- Magnetic pumping transport timescale: 
  $\sim 6$ s (for $T \sim 1$ eV)
Confinement Scales Strongly with Magnetic Field

Not yet dominated by magnetic pumping transport.
Equilibrium Modeling

- Daugherty-Levy Eq.
  \[ \nabla^2 V = \frac{e f(V)}{\varepsilon_0 R^2} \]

- Experimental constraints
  - \( m=1 \) diocotron mode frequency
  \[ f_1 \approx 50 \text{ kHz} \implies \frac{Q}{L} \approx 1.7 \text{ nC/m} \]
  - Central potential on filament
  \[ V_0 \geq -27 \text{ V} \]

- Equilibrium solution:
  - Density \( \sim 0.5 \times 10^7 \text{ cm}^{-3} \)
  - Central potential -23V

3 second confinement time is \(~10^5\) ExB rotations.
Simulating the $m=1$ Mode

Solve Poisson’s equation in toroidal geometry for a uniform density plasma with specified position and radius.

Compute $E_\rho$ and $E_z$ from the potential solution.

Integrate $\mathbf{E}$ along the surface of the electrode sections to obtain the charge on the wall probe.

Calculate the $\mathbf{E} \times \mathbf{B}$ drift at plasma center and update the position and radius of the plasma.
Simulation Results Compared to Data

Signal characteristics used to determine simulation input parameters:
- frequency
- ratio of second harmonic power to fundamental power
Extracting Plasma Parameters using Simulations

**Near-resonant tone burst**
- Excites small amplitude (< 1mm) mode
- Maximizes confinement time.

**Fixed frequency (55 kHz) tone burst**
- Drives mode to larger amplitude
- Incomplete autoresonance \([1,2]\)
- Accelerates charge loss.

\[
\frac{Q}{L} = \frac{2}{\pi} \epsilon_0 L b \left( \frac{1}{1 - \left(\frac{A_1}{b}\right)^2} \right)
\]


Future Work: *Fully Toroidal Trapping*

- Fully Toroidal Injection Phase
- Partial Torus Trapping Phase
- Filament Retraction Phase
- Full Torus Trapping Phase
Pneumatic Filament Retraction System

- Filament mounted on a welded bellows feedthru
- Solenoid activated pneumatic switch drives retraction
- Retraction time ~ 0.1 seconds
Future Work:

*Launch, detect, and model the $m=2$ diocotron mode*

- Frequency of the $m=2$ mode yields information on *density*.
- Coupled with total charge measurement from $m=1$ mode frequency, can get measurement of *transport*.

Simulation results with $m=1$ and $m=2$ mode
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