

Lab 5: Series RLC Resonant Circuit Experiment

Object: To explore the properties of the series RLC circuit – a resonant filter that passes a narrow band of central frequencies while discriminating against all others. This experiment covers capacitance, inductance, reactance, impedance, resonance, the quality factor Q , resonant bandwidth, and loading effects.

Apparatus: HP 200CD Oscillator, Tektronix TDS2004B Digital Oscilloscope, circuit chassis, resistor, capacitor, soldering station.

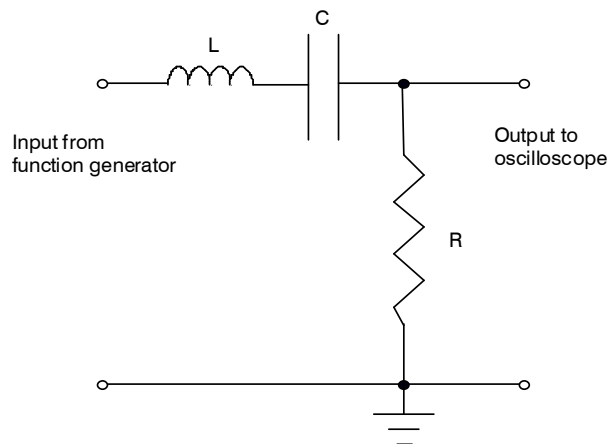


FIG. 1 Ideal RLC series circuit

Introduction: Analysis of the RLC series circuit shown in Fig. 1 yields the following prediction for the ratio of the amplitudes of the output to input voltages,

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + Q_o^2 (f/f_o)^2 (1 - (f_o/f)^2)^2}}, \quad (1)$$

where f is the frequency of the oscillating input and output voltages. The resonant frequency is given by $f_o = 1/2\pi\sqrt{LC}$. The *quality factor* is $Q_o = 2\pi f_o L/R$. If Q_o is large (small), Eq. 1 assumes the form of a narrow (broad) bell-shaped resonance curve centered at the resonant frequency, f_o . The “width” of this curve, *i.e.* the frequency interval between the 0.707 half-power frequencies f_+ and f_- , is

$$\Delta f = f_+ - f_- = \frac{f_o}{Q_o}. \quad (2)$$

Figure 2 shows a more realistic RLC circuit in which the inductor presents an internal resistance R_L as well as inductance. Since the resistances R_S and R_L appear in series, they sum to a total circuit resistance $R_T = R_S + R_L$, and the circuit as a whole exhibits a smaller *total Q* factor $Q_T = 2\pi f_o L/R_T$. It is left as an exercise for the student to show that the input/output expression for this more realistic circuit is

$$\frac{V_{out}}{V_{in}} = \frac{R_S/R_T}{\sqrt{1 + Q_T^2 (f/f_o)^2 (1 - (f_o/f)^2)^2}}. \quad (3)$$

One can derive a prediction for the phase shift between the output and input voltage for the circuit in Fig. 2. It is

$$\phi(f) = \tan^{-1} \left(\frac{1}{2\pi f R_T C} - \frac{2\pi f L}{R_T} \right). \quad (4)$$

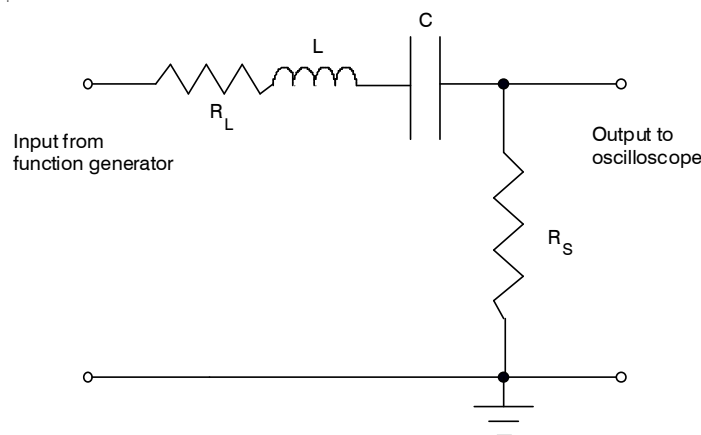


FIG. 2 More realistic RLC series circuit

Measurements:

1. The inductor for this experiment, which is already mounted in the chassis, has an inductance $L \approx 2.0$ H and an intrinsic quality factor $Q_L \equiv \omega L/R_L \approx 44$ (both determined at 1 kHz). These values should be marked on your chassis. Using the values for the inductance and intrinsic quality factor, calculate the “ac” resistance of your inductor at 1 kHz. Then measure the coil’s “dc” resistance at $f=0$ Hz using the Fluke digital multimeter as an ohmmeter. The fact that the “ac” and “dc” resistance for the inductor differ significantly can be understood as follows: as the frequency of the sinusoidal current passing through the coil (inductor) increases, the iron core inside the coil (which helps enhance the inductance) becomes increasingly *lossy*. This energy dissipating effect is a phenomenon that manifests itself as an apparent increase in the effective coil resistance. Since R_L increases roughly linearly with frequency, the coil’s quality factor $Q_L = \omega L/R_L$ remains fairly constant over limited frequency intervals.

2. Construct a RLC series circuit like the one shown in Fig. 1 (and more realistically represented by the circuit in Fig. 2) using a small R_S (about 680Ω) so as to make Q_T fairly large. The capacitor provided to you has a capacitance of $C \approx 0.012 \mu\text{F}$, chosen to produce a resonant frequency near 1 kHz. Convince yourself that this is the case by using the theoretical equation that predicts the resonant frequency. Construct the circuit

in your circuit chassis and have your instructor check your circuit layout and solder technique.

3. Connect the HP 200CD Oscillator to the circuit input and to the CH1 input on the TDS2004B oscilloscope (using a BNC 'T'). Connect the circuit output to the CH2 input on the scope. Set the probe attenuation settings for both input channels to 1X. Trigger on CH1 (AUTO, Rising Slope).

4. Proceed to acquire input-output data that lets you test the theoretical predictions. Your amplitude data should consist of pairs of voltage values (V_{in} , V_{out}) **and** time delay values (to be used to calculate the phase shift) acquired at frequencies ranging from $f_o/50$ to $500f_o$ where f_o is the resonant frequency. Be sure to gather 15 or more data points between the half-power frequencies.

4. Plot V_{out}/V_{in} versus f using Kaleidagraph (or some other graphing program) giving each point an appropriate error bar. Compare the experimental data to the theoretical predictions (Eq. 2) by using Kaleidagraph's nonlinear curve fitting feature with the resonant frequency, quality factor and resistance ratio R_S/R_T as fit parameters. Ask your instructor if you need help writing the fit function. Do the values of the fit parameters agree with the theoretical expectations given the values of the circuit component parameters (within uncertainties)? Determine the experimental half-power frequencies and the experimentally inferred width of the resonance. Compare to the prediction obtained from Eq. 2.

5. Also plot the phase shift versus f in order to test the theoretical prediction from Eq. 4. What is the phase shift at resonance? Explain this using a phasor diagram.

6. If time permits, try repeating the resonance curve measurements with a different capacitor, in order to shift the resonance frequency. Does the quality factor change? Why?