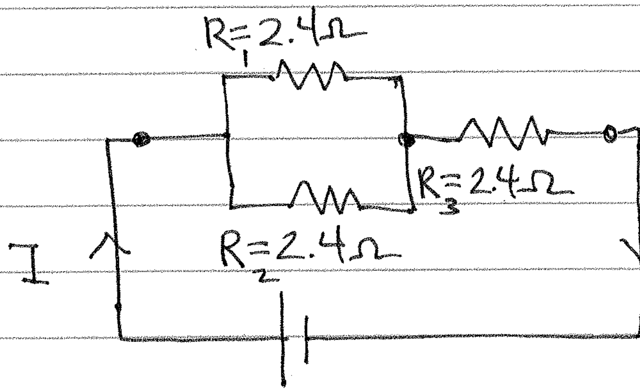
26.58 Each resistor has a resistance of $2.4\,\Omega$ and can dissipate 36 W. What is the maximum total power the circuit can dissipate?



When connected to a source of emf, the entire current I passes through R_3 while only half as much passes through each of the resistors in parallel (R_1 and R_2 ~~due~~ because they are equal).

When R_3 is dissipating the maximum power... $P_3 = I^2R = 36\,W$, then R_1 and R_2 will each dissipate $P_1 = P_2 = \left(\frac{I}{2}\right)^2 R$

The total power dissipated is $P_{TOT} = P_1 + P_2 + P_3 = \left(\frac{I}{2}\right)^2 R + \left(\frac{I}{2}\right)^2 R + I^2R$

$$P_{TOT} = \frac{3}{2} I^2 R \quad \text{or} \ldots \quad \frac{3}{2} \cdot 36\,W = \boxed{54\,W}$$

We do not need to know the current, but can calculate it to be

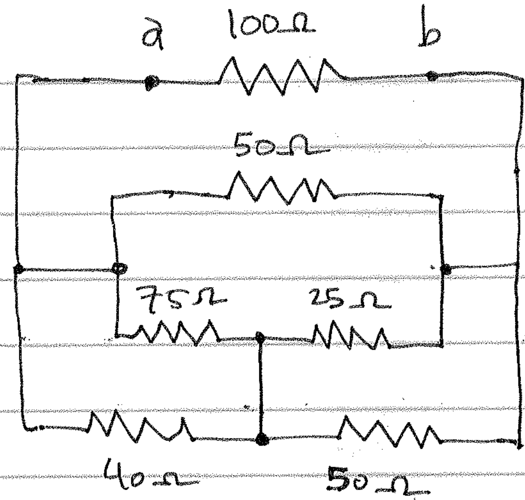
$$I = \sqrt{\frac{P_1}{R_1}} = \sqrt{\frac{36\,W}{2.4\,\Omega}} = \sqrt{15}\,A = 3.9\,A$$

We can also calculate the voltage across the resistor network

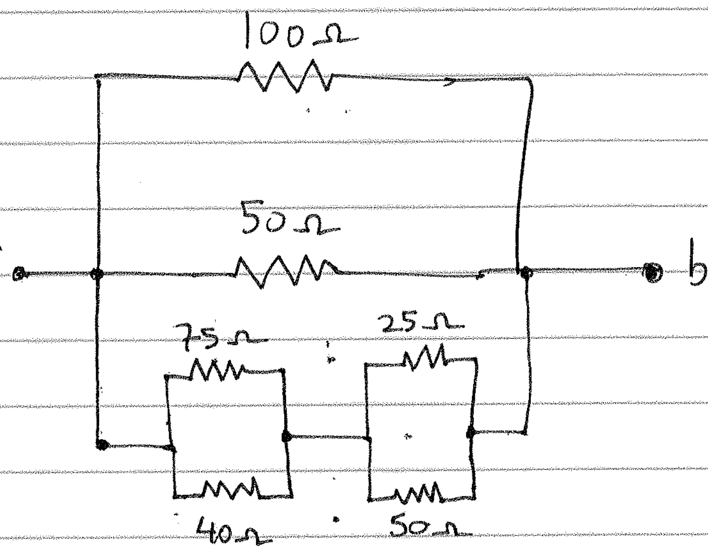
$$P_{TOT} = IV \quad \text{so} \quad V = \frac{54\,W}{3.9\,A} = 13.9\,V$$

26.59 Find the equivalent resistance between points a and b.

(a)



This circuit is equivalent to the following



find these parallel combinations ...

$$\left(\frac{1}{75\,\Omega}+\frac{1}{40\,\Omega}\right)^{-1}=\frac{75\cdot 40\,\Omega^2}{115\,\Omega}=26.1\,\Omega$$

$$\left(\frac{1}{25\,\Omega}+\frac{1}{50\,\Omega}\right)^{-1}=\frac{25\cdot 50\,\Omega^2}{75\,\Omega}=16.7\,\Omega$$

These are combined in series to make

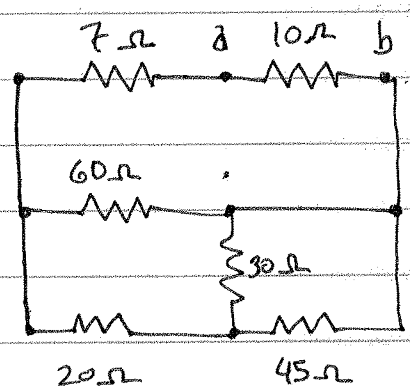
42.8 Ω

Then find the parallel combination of 100 Ω, 50 Ω, and 42.8 Ω

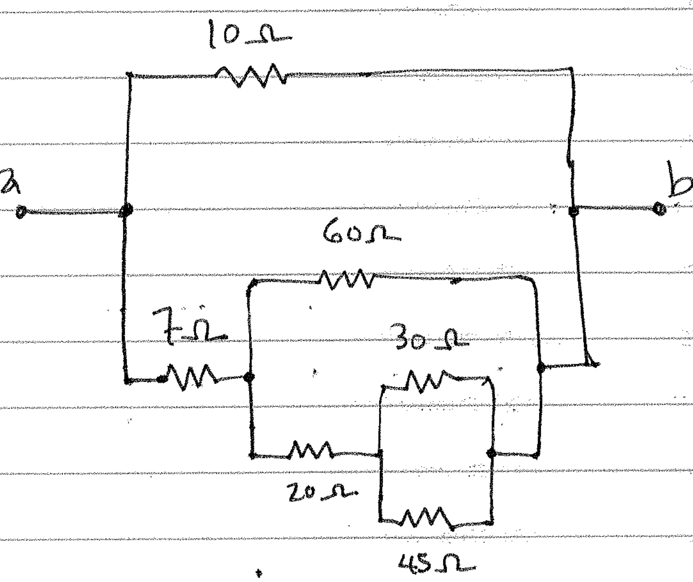
$$\frac{1}{R_{eq}} = \frac{1}{100} + \frac{1}{50} + \frac{1}{42.8} = 0.053\ \Omega^{-1}$$

$$\boxed{R_{eq} = 18.7\ \Omega}$$

(b)



This circuit is equivalent to the following



parallel combination is $\frac{30\,\Omega \cdot 45\,\Omega}{75\,\Omega} = 18\ \Omega$

Which is in series with 20 Ω making 38 Ω

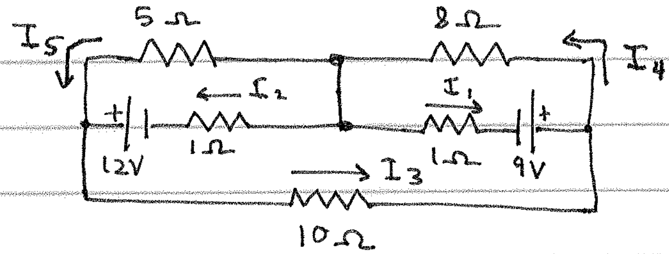
38 Ω is in parallel with 60 Ω ... resulting in $\frac{38\,\Omega \cdot 60\,\Omega}{98\,\Omega} = 23.3\ \Omega$

Which is in series with 7 Ω making 30.3 Ω.

30.3 Ω is in parallel with 10 Ω resulting in an overall resistance of $\frac{30.3 \cdot 10}{40.3} = \boxed{7.52\ \Omega}$

26.61

Calculate the three currents I_1, I_2, and I_3



Define additional currents I_4 and I_5 as shown in diagram

By Kirchoff's junction rule $\boxed{I_4 = I_1 + I_3}$ (1)

and $\boxed{I_5 = I_3 - I_2}$ (2)

Now use Kirchoff's loop rule

upper right loop. $9V - I_4 \cdot 8\Omega - I_1 \, 1\Omega = 0$

or... $9V = 8\Omega \cdot I_4 + I_1 \cdot 1\Omega$ use (1)

or $\boxed{9V = 9 I_1 + 8 I_3}$ (3)

upper left loop $12V + I_5 \cdot 5\Omega - I_2 \cdot 1\Omega = 0$

voltage increases going against current

or $12V = -5\Omega \cdot I_5 + I_2 \cdot 1\Omega$ use (2)

or $\boxed{12V = 6 I_2 - 5 I_3}$ (4)

outer loop $I_3 \cdot 10\Omega + I_5 \cdot 5\Omega + I_4 \cdot 8\Omega = 0$ use (1) and (2)

$10\Omega \cdot I_3 + 5\Omega(I_3 - I_2) + 8\Omega(I_1 + I_3) = 0$

$$\boxed{8 I_1 - 5 I_2 + 23 I_3 = 0}$$ (5)

We now have 3 linear equations in 3 unknowns

Could use matrix inversion technique ...or brute force algebra...

$$\left.\begin{aligned} &\text{From (3)} \quad I_1 = 1 - \frac{8}{9} I_3 \\ &\text{From (4)} \quad I_2 = 2 + \frac{5}{6} I_3 \end{aligned}\right\} \text{subst. into (5)}$$

$$8\left(1 - \frac{8}{9} I_3\right) - 5\left(2 + \frac{5}{6} I_3\right) + 23 I_3 = 0$$

$$-2 + \underbrace{\left(23 - \frac{64}{9} - \frac{25}{6}\right)}_{11.72} I_3 = 0$$

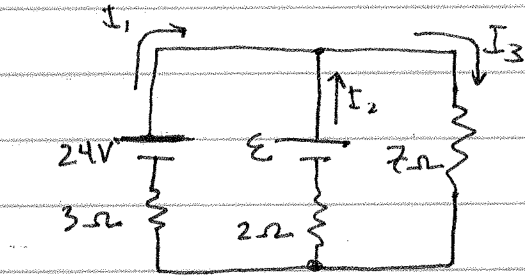
$$I_3 = \frac{2}{11.72} = \boxed{0.17\ A}$$

$$I_1 = 1 - \frac{8}{9} \cdot 0.17A = \boxed{0.85\ A}$$

$$I_2 = 2 + \frac{5}{6} \cdot 0.17 = \boxed{2.14\ A}$$

26.62

Find emf such that there is 1.80 A through the 7Ω resistor in the following circuit.



By Kirchoff's junction rule

$$I_3 = I_1 + I_2 = 1.80\text{ A}$$

Kirchoff's Loop rule:

Outer loop $\quad -3\Omega \cdot I_1 + 24\text{V} - \underbrace{I_3 \cdot 7\Omega}_{12.6\text{V}} = 0$

So... $I_1 = \dfrac{(24 - 12.6)}{3} = \underline{3.80\text{ A}} \rightarrow I_2 = I_3 - I_1 = -2.00\text{ A}$

down... changing $\mathcal{E}$

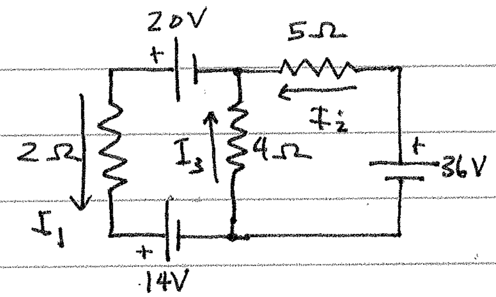
Right hand loop.

$$\mathcal{E} - I_3 \cdot 7\Omega - I_2 \cdot 2\Omega = 0$$

$$\mathcal{E} = \underbrace{7 \cdot 1.80\text{ A}}_{12.6\text{V}} - \underbrace{2.00\text{ A} \cdot 2\Omega}_{4\text{V}}$$

$$\boxed{\mathcal{E} = 8.60\text{V}}$$

26.63 Find the current through each of the three resistors in the following circuit.



Choose the current directions as shown at the left. If one or more turn out to be negative, the current is actually in the other direction.

Kirchoff's junction rule: $I_1 = I_2 + I_3$ (1)

Kirchoff's loop rule: Outer loop $36V - I_2 \cdot 5\Omega + 20 - I_1 \cdot 2\Omega - 14V = 0$

or $42V - 5I_2 - 2I_1 = 0$ (2)

Lefthand loop $20V - I_1 \cdot 2\Omega - 14V - I_3 \cdot 4\Omega = 0$

or $6V - 4I_3 - 2I_1 = 0$ (3)

Three equations and three unknowns

Subst. (1) into (2) and (3)

$$42 - 5I_2 - 2(I_2 + I_3) = 0$$

or $42 - 7I_2 - 2I_3 = 0$ (4)

and $6 - 4I_3 - 2(I_2 + I_3) = 0$

or $6 - 2I_2 - 6I_3 = 0$ (5)

$[3 \times (4)] - (5) \longrightarrow$ $120 - 19I_2 = 0$

$$I_2 = \frac{120}{19}\text{ Amps} = \boxed{6.32\text{ A}}$$

from (4)

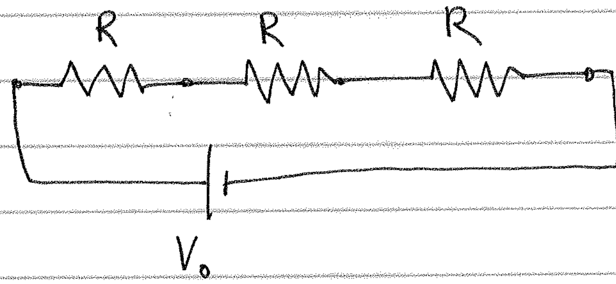
$$I_3 = 21 - \frac{7}{2}I_2 = 21 - \frac{7}{2}\cdot 6.32 = \boxed{-1.11\text{A}}$$

current goes down thru 4Ω resistor

$$I_1 = I_2 + I_3 = \boxed{5.21\text{ A}}$$

26.70

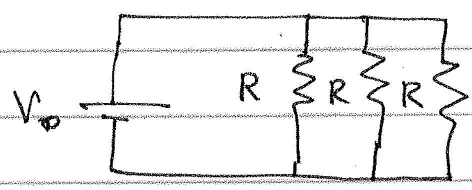
Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 27 W.



$R_{eq} = 3R$

$$P_{series} = \frac{V_0^2}{R_{eq}} = \frac{V_0^2}{3R} = 27\,W$$

What power would be dissipated if the 3 resistors were connected in parallel across the same potential difference?



$$R_{eq} = \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1}$$

$$R_{eq} = \frac{R}{3}$$

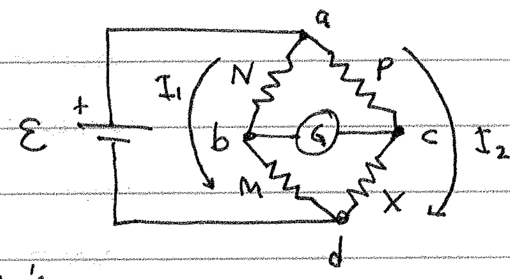
$$P_{parallel} = \frac{V_0^2}{(R/3)} = \frac{3V_0^2}{R} = 9P_{series}$$

$$\boxed{P_{parallel} = 243\,W}$$

26.79

The Wheatstone Bridge.

(a) Find balanced condition



If there is no deflection of the galvanometer, then $V_b = V_c$

And current in N = current in M, $\rightarrow I_1$

Or.. $V_a - V_b = V_a - V_c \xrightarrow[\text{Law}]{\text{ohm's}} I_1 N = I_2 P \quad (1)$

And current in P = current in X $\rightarrow I_2$

and $V_b - V_d = V_c - V_d \longrightarrow I_1 M = I_2 X \quad (2)$

Divide ~~(1) ÷ (2)~~ $(2) \div (1) \longrightarrow \dfrac{X}{P} = \dfrac{M}{N}$ or $\boxed{X = \dfrac{MP}{N}}$

(b) If galvanometer reads zero (bridge is balanced) and $M = 850.0\,\Omega$, $N = 15.00\,\Omega$ and $P = 33.48\,\Omega$, what is X?

$$X = \frac{MP}{N} = \frac{850 \cdot 33.48}{15} = \boxed{1897\,\Omega}$$