2.4 The Lorentz Transformation

10. Use the spherical wavefronts of Equations (2.9) to derive the Lorentz transformation given in Equations (2.17). Supply all the steps.

11. Show that both Equations (2.17) and (2.18) reduce to the Galilean transformation when $v \ll c$.

12. Determine the ratio $\beta = v/c$ for the following: (a) A car traveling 100 km/h. (b) A commercial jet airliner traveling 290 m/s. (c) A supersonic airplane traveling at Mach 2.3 (Mach number $= v/v_{\text{sound}}$). (d) The space shuttle, traveling 27,000 km/h. (e) An electron traveling 25 cm in 2 ns. (f) A proton traveling across a nucleus (10^{-14} m) in 0.35×10^{-22} s.

13. Two events occur in an inertial system K as follows:

Event 1: $x_1 = a, \quad t_1 = 2a/c, \quad y_1 = 0, \quad z_1 = 0$

Event 2: $x_2 = 2a, \quad t_2 = 3a/2c, \quad y_2 = 0, \quad z_2 = 0$

In what frame K′ will these events appear to occur at the same time? Describe the motion of system K′.

14. An event occurs in system K′ at $x' = 2$ m, $y' = 3.5$ m, $z' = 3.5$ m, and $t' = 0$. System K′ and K have their axes coincident at $t = t' = 0$, and system K′ travels along the x axis of system K with a speed $0.8c$. What are the coordinates of the event in system K?

15. A light signal is sent from the origin of a system K at $t = 0$ to the point $x = 3$ m, $y = 5$ m, $z = 10$ m. (a) At what time t is the signal received? (b) Find (x', y', z', t') for the receipt of the signal in a frame K′ that is moving along the x axis of K at a speed of $0.8c$. (c) From your results in (b) verify that the light traveled with a speed c as measured in the K′ frame.

16. Is there a frame K′ in which the two events described in Problem 13 occur at the same place? Explain.

17. Find the relativistic factor γ for each of the parts of Problem 12.

2.5 Time Dilation and Length Contraction

18. Show that the experiment depicted in Figure 2.11 and discussed in the text leads directly to the derivation of length contraction.

19. A rocket ship carrying passengers blasts off to go from New York to Los Angeles, a distance of about 5000 km. How fast must the rocket ship go to have its own length shortened by 1%?

20. Astronomers discover a planet orbiting around a star similar to our sun that is 20 lightyears away. How fast must a rocket ship go if the round trip is to take no longer than 40 years in time for the astronauts aboard? How long will the trip take as measured on Earth?

21. Particle physicists use particle track detectors to determine the lifetime of short-lived particles. A muon has a mean lifetime of 2.2 μs and makes a track 9.5 cm long before decaying into an electron and two neutrinos. What was the speed of the muon?

22. The Apollo astronauts returned from the moon under the Earth's gravitational force and reached speeds of almost 25,000 mi/h with respect to Earth. Assuming (incorrectly) they had this speed for the entire trip from the moon to Earth, what was the time difference for the trip between their clocks and clocks on Earth?

23. A clock in a spaceship is observed to run at a speed of only 3/5 that of a similar clock at rest on Earth. How fast is the spaceship moving?

24. A spaceship of length 40 m at rest is observed to be 20 m long when in motion. How fast is it moving?

25. The Concorde traveled 8000 km between two places in North America and Europe at an average speed of 375 m/s. What is the total difference in time between two similar atomic clocks, one on the airplane and one at rest on Earth during a one-way trip? Consider only time dilation and ignore other effects like the rotation of the Earth. .

26. A mechanism on Earth used to shoot down geosynchronous satellites that house laser-based weapons is finally perfected and propels golf balls at $0.94c$. (a) How far will a detector placed inside the golf ball initially measure the distance to the satellite? (Geosynchronous satellites are placed 3.58×10^4 km above the surface of the Earth.) (b) How much time will it take the golf ball to make the journey to the satellite in the Earth's frame? How much time will it take in the golf ball's frame?

27. Two events occur in an inertial system K at the same time but 4 km apart. What is the time difference measured in a system K′ moving between these two events when the distance separation of the events is measured to be 5 km?

28. Imagine that in another universe the speed of light is only 100 m/s. (a) A person traveling along an interstate highway at 120 km/h ages how much slower than a person at rest? (b) This person passes by a meterstick at rest on the highway. How long does the meterstick appear?

29. In another universe where the speed of light is only 100 m/s, an airplane that is 40 m long at rest and flies at 300 km/h will appear to be how long to an observer at rest?

30. Two systems K and K′ synchronize their clocks at $t = t' = 0$ when their origins are aligned as system K′ passes by system K along the x axis at relative speed $0.8c$. At time $t = 3$ ns, Frank (in system K) shoots a proton gun having proton speeds of $0.98c$ along his x axis. The protons leave the gun at $x = 1$ m and arrive at a target 120 m away. Determine the event coordinates (x, t) of the gun firing and of the protons arriving as measured by observers in both systems K and K′.

2.6 Addition of Velocities

31. A spaceship is moving at a speed of $0.8c$ away from an observer at rest. A boy in the spaceship shoots a proton gun with protons having a speed of $0.7c$. What is the speed of the protons measured by the observer at rest when the gun is shot (a) away from the observer and (b) toward the observer?

A proton and an antiproton are moving toward each other in a head-on collision. If each has a speed of $0.8c$ with respect to the collision point, how fast are they moving with respect to each other?

33. Imagine the speed of light in another universe to be only 100 m/s. Two cars are traveling along an interstate highway in opposite directions. Person 1 is traveling 110 km/h, and person 2 is traveling 140 km/h. How fast does person 1 measure person 2 to be traveling? How fast does person 2 measure person 1 to be traveling?

34. In the Fizeau experiment described in Example 2.5, suppose that the water is flowing at a speed of 5 m/s. Find the difference in the speeds of two beams of light, one traveling in the same direction as the water and the other in the opposite direction. Use $n = 1.33$ for water.

35. Three galaxies are aligned along an axis in the order A, B, C. An observer in galaxy B is in the middle and observes that galaxies A and C are moving in opposite directions away from him, both with speeds $0.60c$. What is the speed of galaxies B and C as observed by someone in galaxy A?

36. Consider the *gedanken* experiment discussed in Section 2.6 in which a giant floodlight stationed 400 km above the Earth's surface shines its light across the moon's surface. How fast does the light flash across the moon?

2.7 Experimental Verification

37. A group of scientists decide to repeat the muon decay experiment at the Mauna Kea telescope site in Hawaii, which is 4205 m (13,796 feet) above sea level. They count 10^4 muons during a certain time period. Repeat the calculation of Section 2.7 and find the classical and relativistic number of muons expected at sea level. Why did they decide to count as many as 10^4 muons instead of only 10^3?

38. Consider a reference system placed at the U.S. Naval Observatory in Washington, D.C. Two planes take off from Washington Dulles airport, one going eastward and one going westward, both carrying a cesium atomic clock. The distance around the Earth at 39° latitude (Washington, D.C.) is 31,000 km, and Washington rotates about the Earth's axis at a speed of 360 m/s. Calculate the predicted differences between the clock left at the observatory and the two clocks in the airplanes (each traveling at 300 m/s) when the airplanes return to Washington. Include the rotation of the Earth but no general relativistic effects. Compare with the predictions given in the text.

2.8 Twin Paradox

39. Derive the results in Table 2.1 for the frequencies f' and f''. During what time period do Frank and Mary receive these frequencies?

40. Derive the results in Table 2.1 for the time of the total trip and the total number of signals sent in the frame of both twins.

2.9 Spacetime

41. Using the Lorentz transformation, prove that $s^2 = s'^2$.

42. Prove that for a timelike interval, two events can never be considered to occur simultaneously.

43. Prove that for a spacelike interval, two events cannot occur at the same place in space.

44. Given two events, (x_1, t_1) and (x_2, t_2), use a spacetime diagram to find the speed of a frame of reference in which the two events occur simultaneously. What values may Δs^2 have in this case?

45. (a) Draw on a spacetime diagram in the fixed system a line expressing all the events in the moving system that occur at $t' = 0$ if the clocks are synchronized at $t = t' = 0$. (b) What is the slope of this line? (c) Draw lines expressing events occurring for the four times t'_4, t'_3, t'_2, and t'_1 where $t'_4 < t'_3 < 0 < t'_2 < t'_1$. (d) How are these four lines related geometrically?

46. Consider a fixed and a moving system with their clocks synchronized and their origins aligned at $t = t' = 0$. (a) Draw on a spacetime diagram in the fixed system a line expressing all the events occurring at $t' = 0$. (b) Draw on this diagram a line expressing all the events occurring at $x' = 0$. (c) Draw all the worldlines for light that pass through $t = t' = 0$. (d) Are the x' and ct' axes perpendicular? Explain.

47. Use the results of the two previous problems to show that events simultaneous in one system are not simultaneous in another system moving with respect to the first. Consider a spacetime diagram with x, ct and x', ct' axes drawn such that the origins coincide and the clocks were synchronized at $t = t' = 0$. Then consider events 1 and 2 that occur simultaneously in the fixed system. Are they simultaneous in the moving system?

2.10 Doppler Effect

48. An astronaut is said to have tried to get out of a traffic violation for running a red light ($\lambda = 670$ nm) by telling the judge that the light appeared green ($\lambda = 540$ nm) to her as she passed by in her high-powered transport. If this is true, how fast was the astronaut going?

49. Derive Equation (2.32) for the case where the source is fixed, but the receiver approaches it with velocity v.

50. Do the complete derivation for Equation (2.33) when the source and receiver are receding with relative velocity v.

51. A spacecraft traveling out of the solar system at a speed of $0.92c$ sends back information at a rate of 400 Hz. At what rate do we receive the information?

52. Three radio-equipped plumbing vans are broadcasting on the same frequency f_0. Van 1 is moving east of van 2 with speed v, van 2 is fixed, and van 3 is moving

81. Protons of 10^{12} eV (TeV) energy are produced at the Fermilab. Calculate the speed, momentum, and total energy of the protons.

82. What is the kinetic energy of (a) an electron having a momentum of 30 GeV/c? (b) a proton having a momentum of 30 GeV/c?

83. A muon has a mass of 106 MeV/c^2. Calculate the speed, momentum, and total energy of a 200-MeV muon.

84. The reaction ^{2}H + ^{2}H $\rightarrow n$ + ^{3}He (where n is a neutron) is one of the reactions useful for producing energy through nuclear fusion. (a) Assume the deuterium nuclei (^{2}H) are at rest and use the atomic mass units of the masses in Appendix 8 to calculate the mass-energy imbalance in this reaction. (Note: you can use atomic masses for this calculation, because the electron masses cancel out.) This amount of energy is given up when this nuclear reaction occurs. (b) What percentage of the initial rest energy is given up?

85. The reaction ^{2}H + ^{3}H $\rightarrow n$ + ^{4}He is one of the reactions useful for producing energy through nuclear fusion. (a) Assume the deuterium (^{2}H) and tritium (^{3}H) nuclei are at rest and use the atomic mass units of the masses in Appendix 8 to calculate the mass-energy imbalance in this reaction. This amount of energy is given up when this nuclear reaction occurs. (b) What percentage of the initial rest energy is given up?

2.14 Electromagnetism and Relativity

86. Instead of one positive charge outside a conducting wire, as was discussed in Section 2.14 and shown in Figure 2.34, consider a second conducting wire parallel to the first one. Both wires have positive and negative charges, and the wires are electrically neutral. Assume that in both wires the positive charges travel to the right and negative charges to the left. (a) Consider an inertial frame moving with the negative charges of wire 1. Show that the second wire is attracted to the first wire in this frame. (b) Now consider an inertial frame moving with the positive charges of the second wire. Show that the first wire is attracted to the second. (c) Use this argument to show that electrical and magnetic forces are relative.

General Problems

An Ω^- particle has rest energy 1672 MeV and mean lifetime 8.2×10^{-11} s. It is created and decays in a particle track detector and leaves a track 24 mm long. What is the total energy of the Ω^- particle?

88. Show that the following form of Newton's second law satisfies the Lorentz transformation. Assume the force is parallel to the velocity.

$$F = m\frac{dv}{dt}\frac{1}{[1 - (v^2/c^2)]^{3/2}}$$

89. Using the results listed in Table 2.1, find the time of detecting Mary's turnaround and the number of signals received at the rate f'.

90. Using the previous results listed in Table 2.1, find the time for the remainder of the trip, the number of signals received at the rate f'', and the total number of signals received. Determine the conclusion as to the other twin's measure of the time taken for the trip.

91. Frank and Mary are twins. Mary jumps on a spaceship and goes to Alpha Centauri (4 lightyears away) and returns. She travels at a speed of 0.8c with respect to Earth and emits a radio signal every week. Frank also sends out a radio signal to Mary once a week. (a) How many signals does Mary receive from Frank before she turns around? (b) At what time does the frequency of signals Frank receives suddenly change? How many signals has he received at this time? (c) How many signals do Frank and Mary receive for the entire trip? (d) How much time does the trip take according to Frank and to Mary? (e) How much time does each twin say the other twin will measure for the trip? Do the answers agree with those for (d)?

An electron has a total energy that is 200 times its rest energy. Determine its (a) kinetic energy, (b) speed, and (c) momentum.

A proton moves with a speed of 0.9c. Find the speed of an electron that has (a) the same momentum as the proton, and (b) the same kinetic energy.

A high-speed K^0 meson is traveling at a speed of 0.9c when it decays into a π^+ and a π^- meson. What are the greatest and least speed that the mesons may have?

95. Frank and Mary are twins, and Mary wants to travel to our nearest star, the binary star Alpha Centauri (4.3 lightyears away). Mary leaves on her 30th birthday and intends to return to Earth on her 52nd birthday. (a) Assuming her spaceship returns from Alpha Centauri without stopping, how fast must her spaceship travel? (b) How old will Frank be when she returns?

96. The International Space Federation constructs a new spaceship that can travel at a speed of 0.995c. Mary, the astronaut, boards the spaceship to travel to Barnard's star, which is the second nearest star to our solar system after Alpha Centauri and is 5.98 ly away. After reaching Barnard's star, the spaceship travels slowly around the star system for 3 years doing research before returning back to Earth. (a) How much time does her journey take? (b) How much older is her twin Frank when she returns?

97. A powerful laser on Earth rotates its laser beam in a circle at a frequency of 0.03 Hz. (a) How fast does the spot that the laser makes on the moon move across the moon's landscape? (b) With what rotation frequency should the laser rotate if the laser spot moves across the moon's landscape at speed c?