SOLUTIONS

PHYS160 Spring 2009

Problem Set #5: Due in class on Wed. 5/13

Problems from Chapter 7 of Thornton & Rex: 19, 22, 23, 30, 34

Problems from Chapter 8 of Thornton & Rex: 5, 10, 12, 15, 24, 26

$\boxed{7-23}$ For a hydrogen atom in the 6f state, what is the minimum angle between the orbital angular momentum vector and the z-axis?

The smallest angle between $\vec{L}$ and the z-axis corresponds to the largest value of $m_\ell = +\ell$. So the z-component of $\vec{L}$ is... $L_z = \ell\hbar$ and the angle with respect to the z-axis is...

$$\theta_{min} = \cos^{-1}\left(\frac{L_{z,max}}{|\vec{L}|}\right) = \cos^{-1}\left(\frac{\ell}{\sqrt{\ell(\ell+1)}}\right)$$

For 6f state ... $\ell = 3$ so...

$$\theta_{min} = \cos^{-1}\left(\frac{3}{\sqrt{12}}\right) = \boxed{30°}$$

$\boxed{7-30}$ Using all four quantum numbers, write down all possible sets of quantum numbers for the 6d state of hydrogen.

$6d \Leftrightarrow n=6,\ \ell=2$

$$\left.\begin{array}{ll} m_\ell = +2 & m_s = \pm\frac{1}{2} \\ m_\ell = +1 & m_s = \pm\frac{1}{2} \\ m_\ell = 0 & m_s = \pm\frac{1}{2} \\ m_\ell = -1 & m_s = \pm\frac{1}{2} \\ m_\ell = -2 & m_s = \pm\frac{1}{2} \end{array}\right\} \text{10 states}$$

7-19 Calculate the possible z components of the orbital angular momentum for an electron in a 4p state.

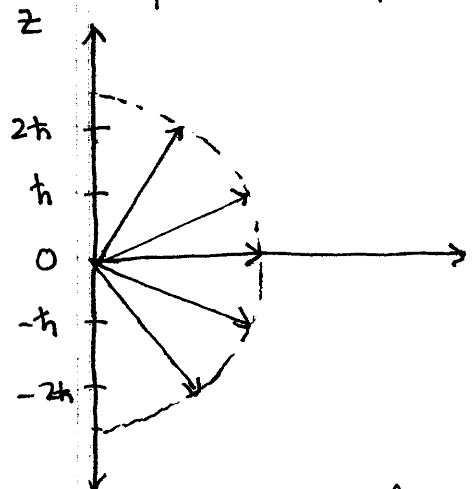
4p refers to the states with $n=4$, $\ell=1$. The z-component of $\vec{L}$ is quantized as...

$$L_z = \hbar m_\ell \quad \text{where } m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$$

When $\ell = 1$ $\quad m_\ell = -1, 0, +1$ $\quad$ so...

$$\boxed{L_z = -\hbar, 0, \hbar}$$

7-22 For hydrogen atoms in a d-state, sketch the orbital angular momentum with respect to the z-axis. Calculate the allowed angles of μ_ℓ with respect to the z-axis.

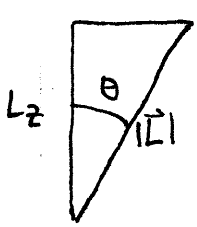


$$|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar \quad \text{for } \ell = 2 \quad (\text{d-state})$$
$$= \sqrt{6}\,\hbar = 2.45\,\hbar$$

For orbital angular momentum of an electron:

$$\vec{\mu} = \frac{-e}{2m}\vec{L} \quad (\text{Eq. } 7.26)$$

so $\vec{\mu}$ is antiparallel to $\vec{L}$ and $\vec{\mu}_\ell$ can have the same angles with respect to the z-axis as $\vec{L}$.



$$\cos\theta = \frac{L_z}{|\vec{L}|} \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{L_z}{|\vec{L}|}\right)$$

← angles that $\vec{\mu}$ makes with respect to z axis for corresponding values of m_ℓ.

m_ℓ	θ	$180° - \theta$
+2	$\cos^{-1}\left(\frac{2}{\sqrt{6}}\right) = 35.3°$	144.7°
+1	$\cos^{-1}\left(\frac{1}{\sqrt{6}}\right) = 65.9°$	114.1°
0	$\cos^{-1}\left(\frac{0}{\sqrt{6}}\right) = 90.0°$	90.0°
−1	$\cos^{-1}\left(-\frac{1}{\sqrt{6}}\right) = 114.1°$	65.9°
−2	$\cos^{-1}\left(-\frac{2}{\sqrt{6}}\right) = 144.7°$	35.3°

$\boxed{7\text{-}34}$ Find whether the following transitions are allowed, and if they are, find the energy involved and whether the photon is absorbed or emitted for the hydrogen atom:

(a) $(5, 2, 1, \frac{1}{2}) \rightarrow (5, 2, 1, -\frac{1}{2})$

These two states are <u>degenerate</u> in the Schrödinger Eq. solution

(b) $(4, 3, 0, \frac{1}{2}) \rightarrow (4, 2, 1, -\frac{1}{2})$

These two states are <u>degenerate</u> in the Schrödinger Eq. solution

(c) $(5, 2, -2, -\frac{1}{2}) \rightarrow (1, 0, 0, -\frac{1}{2})$

$\underline{\underline{\Delta l = -2}}$

This transition is <u>forbidden</u> since it violates the selection rule $\Delta l = \pm 1$.

(d) $(2, 1, 1, \frac{1}{2}) \rightarrow (4, 2, 1, \frac{1}{2})$

This transition is <u>allowed</u> since $\Delta l = +1$ and $\Delta m_l = 0$

Since the energy depends only on n, the transition requires <u>absorption</u> of a photon with energy ...

$$E_\gamma = \Delta E = \frac{-E_0}{n_f^2} - \frac{-E_0}{n_i^2} = \frac{-13.6\,eV}{16} + \frac{13.6\,eV}{4}$$

$$= 13.6\,eV\left[\frac{1}{4} - \frac{1}{16}\right] = 13.6\,eV\left[\frac{3}{16}\right]$$

$$\boxed{\Delta E = 2.55\,eV}$$

$\boxed{8-5}$ Using table 8.1 and fig. 8.2 write down the electron configuration of the following elements: potassium, vanadium, selenium, zirconium, samarium, and uranium.

Potassium: $Z = 19$ ← # of electrons

ground state electron configuration: $\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^1}$

Vanadium: $Z = 23$

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^3}$

Selenium: $Z = 34$

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^4}$

Zirconium: $Z = 40$

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^2}$

Samarium: $Z = 62$

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^6}$

Uranium: $Z = 92$

$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 \underbrace{7s^2 5f^4}}$

Note a slight discrepancy between this configuration obtained using table 8.1 and the configuration listed in fig. 8.2

$[Rn]\ 7s^2 5f^3 6d^1$

$\boxed{8-10}$ What are the electronic configurations of the elements Pd, Hf, and Sb?

Palladium (Pd): $Z = 46$

Using ~~table~~ table 8.1, the ground state electron configuration should be...

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^8}$$

BUT... according ~~to~~ to Fig. 8.2, this element actually has a configuration

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 4d^{10}}$$ with no 5s electrons!

Hafnium (Hf): $Z = 72$

Using table 8.1, the ground state electron configuration should be..

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^2}$$

which agrees with Fig. 8.2

Antimony (Sb): $Z = 51$

Using table 8.1 the ground state electron configuration should be...

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^3}$$

Which agrees with Fig. 8.2

$\boxed{8\text{-}12}$ If the zirconium atom ground state has $S=1$ and $L=3$, what are the permissible values of J?

J can take on values from $|L-S|$ to $(L+S)$ integer units... so

$J = 2, 3, 4$ are possible

Spectroscopic notation is.... $\boxed{^{3}D_2,\ ^{3}D_3,\ ^{3}D_4}$

J

L

$2S+1$

The ground state is likely to be the one with the smallest value of J... $\underline{^{3}D_2}$.

$\boxed{8\text{-}15}$ For the hydrogen atom in the 3d excited state, find the possible values of l, m_l, j, s, m_s, m_j. Give the term notation for each configuration.

The 3d state means $n=3$, $l=2$ .... also $s=\frac{1}{2}$ for hydrogen (one electron)... so $j = 2-\frac{1}{2} = \frac{3}{2}$ or $j = 2+\frac{1}{2} = \frac{5}{2}$

$j=\frac{3}{2}$ ... $m_j = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$ notation $\boxed{3^{2}D_{3/2}}$

$j=\frac{5}{2}$ .... $m_j = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}, +\frac{5}{2}$ notation $\boxed{3^{2}D_{5/2}}$

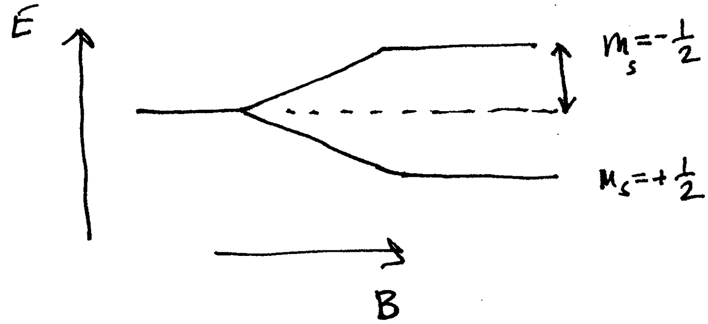
Note that when spin is included, the quantum numbers m_l, and m_s are replaced with j and m_j.

$\boxed{8\text{-}24}$ What is the energy difference between a spin-up state and a spin-down state for an electron in an s state if the magnetic field is 1.7 T?

In an s-state $l = 0$, so the total angular momentum is $j = s = \frac{1}{2}$

And the z-component magnetic moment is... $\mu_z = g_s \mu_B m_s$ (see eq. 7.34)

with $g_s \simeq -2.0$



$$\Delta E = \cancel{E_{up}}\ E_{down} - E_{up} = g_s \mu_B \left(-\tfrac{1}{2}\right) B_{ext} - g_s \mu_B \left(+\tfrac{1}{2}\right) B_{ext}$$

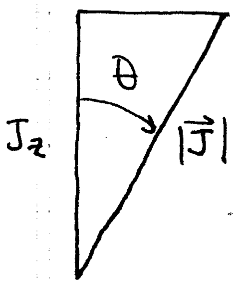
$$= -g_s \mu_B B_{ext}$$

$$= +2.0 \times (5.7884 \times 10^{-5}\ eV/T) \times (1.7\,T)$$

$$\boxed{\Delta E = 1.97 \times 10^{-4}\ eV}$$

8-26 If the minimum angle between the total angular momentum vector and the z-axis is 32.3°, what is the total angular momentum quantum number?

The minimum angle between $\vec{J}$ and the z-axis corresponds to $m_j = j$.



$$\theta_{min} = \cos^{-1}\left(\frac{J_{z,max}}{|\vec{J}|}\right)$$

where $J_{z,max} = m_{j,max}\hbar = j\hbar$

and $|\vec{J}| = \sqrt{j(j+1)}\,\hbar$

So... $$\theta_{min} = \cos^{-1}\left(\frac{j}{\sqrt{j(j+1)}}\right)$$

or... $$\cos\theta_{min} = \frac{j}{\sqrt{j(j+1)}}$$

or... $$\sqrt{j(j+1)}\cos\theta_{min} = j \Rightarrow \frac{\sqrt{j(j+1)}}{j} = \frac{1}{\cos\theta_{min}}$$

square both sides.. $$\frac{j(j+1)}{j^2} = \frac{1}{\cos^2\theta_{min}} \Rightarrow \frac{j^2+j}{j^2} = \frac{1}{\cos^2\theta_{min}}$$

$$1 + \frac{1}{j} = \frac{1}{\cos^2\theta_{min}} \quad \text{or..} \quad \frac{1}{j} = \frac{1}{\cos^2\theta_{min}} - 1$$

$$j = \left(\frac{1}{\cos^2\theta_{min}} - 1\right)^{-1} = \left(\frac{1}{\cos^2 32.3°} - 1\right)^{-1} = 2.5$$

$$\boxed{j = 5/2}$$