SOLUTIONS

PHYS160 Spring 2009

Problem Set #6: Due in class on Wed. 5/27

Problems from Chapter 12 of Thornton & Rex: 4, 6, 12, 13, 17, 24, 26, 27, 30, 37,

Extra Problems:

A: Use the information in Appendix 8 to identify all of the nuclei in the (4n+3) or "Actinium" decay series (see Table 12.3) from U-235 to Pb-207. Make a graph like the one in Fig. 12.16 for this series. (Note: In class I use a convention where the axes are reversed for such diagrams. Please follow my convention and put Z on the vertical axis and N on the horizontal axis).

B: Write down the decay reactions for the following unstable particles and calculate the decay energy.

(i) Radium-226 (alpha decay)
(ii) Potassium-40 (negative beta decay)
(iii) Sodium-22 (positive beta decay)
(iv) Cobalt-57 (electron capture)

$\boxed{12\text{-}4}$ What are the number of neutrons and protons for the following nuclei:

^{6}Li Atomic number $Z = \boxed{3 \text{ protons}}$

Mass number $A = N + Z = 6$

So ... $N = \cancel{A} - Z = 6 - 3 = \boxed{3 \text{ neutrons}}$

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^{13}C $Z = \boxed{6 \text{ protons}}$

$N = 13 - 6 = \boxed{7 \text{ neutrons}}$

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^{40}K $Z = \boxed{19 \text{ protons}}$

$N = 40 - 19 = \boxed{21 \text{ neutrons}}$

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^{64}Cu $Z = \boxed{29 \text{ protons}}$

$N = 64 - 29 = \boxed{35 \text{ neutrons}}$

$\boxed{12\text{-}6}$ Write down the nuclidic symbol and percentage abundances for all the nuclides having atomic numbers 7, 23, and 37.

$Z=7$ Nitrogen: Appendix 8 lists 11 isotopes: ${}^{12}_{7}N_{5}$ through ${}^{22}_{7}N_{15}$

but only two stable/naturally occuring isotopes:

$${}^{14}_{7}N_{7}\ (99.63\%) \quad \text{and} \quad {}^{15}_{7}N_{8}\ (0.37\%)$$

$Z=23$ Vanadium: Appendix 8 lists 8 isotopes ${}^{47}_{23}V_{24}$ through ${}^{54}_{23}V_{31}$

with two of these occuring naturally:

$${}^{51}_{23}V_{28}\ (99.750\%) \quad \text{and} \quad {}^{50}_{23}V_{27}\ (0.250\%)$$

with the latter being an unstable isotope with an exceedingly long half-life of 1.4×10^{17} y (electron capture or β^- decay).

$Z=37$ Rubidium: 17 isotopes ${}^{75}_{37}Rb_{38}$ through ${}^{91}_{37}Rb_{54}$

Two naturally occurring isotopes

$${}^{85}_{37}Rb_{48}\ (72.17\%) \quad \text{and} \quad {}^{87}_{37}Rb_{50}\ (27.83\%)$$

unstable β^- emitter with half-life of 4.75×10^{10} y

12-12 Threshold energy for photodisintegration of a deuteron:

Equation 12.14 on p.432 gives the threshold energy, including conservation of momentum, for a deuteron at rest.

$$E_{min} = B_d \left[1 + \frac{B_d}{2Mc^2} \right]$$

where $B_d = 2.224$ MeV (Eq. 12.9)

and $Mc^2 = 2.014102\,u \times 931.494043 \frac{MeV}{uc^2} \times c^2 = 1876$ MeV

↳ rest mass energy of deuteron.

So...

$$E_{min} = 2.224 \text{ MeV} \left[1 + \frac{2.224}{2 \times 1876} \right] = 2.224 \left[1.00056 \right]$$

$$\boxed{E_{min} = 2.225 \text{ MeV}}$$

This differs from the binding energy very little. If momentum conservation were ignored, we would get $E'_{min} = B_d$

The percentage error in that calculation would be...

$$\frac{\%\,\text{error}}{100} = \frac{E_{min} - E'_{min}}{E'_{min}} \text{ or } \frac{E_{min} - B_d}{B_d} = \frac{B_d}{2 \cdot Mc^2} = 0.00056$$

or... $\boxed{\%\,\text{error} = 0.056\%}$ very small.

$\boxed{12\text{-}13}$ Compute the gravitational and Coulomb force between two protons in ^3He.

The average nuclear potential is attractive and ~40MeV and the nuclear radius is ~3.0 fm. Compare the nuclear force to the other two.

* Gravitational force: $F_g = \frac{G m_1 m_2}{r^2}$

$m_1 = m_2 = m_p = 1.67\times10^{-27}$ kg

$G = 6.67\times10^{-11}$ Nm^2/kg^2

$r = 3\times10^{-15}$ m

So.. $F_g = \frac{6.67\times10^{-11}\cdot(1.67\times10^{-27})^2}{(3\times10^{-15})^2} = \frac{6.67\cdot1.67^2}{9}\times10^{-11}\cdot10^{-54}\cdot10^{30}$ N

$$\boxed{F_g \cong 2\times10^{-35}\ N}$$

* Coulomb force $F_e = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r^2}$

$e = 1.60\times10^{-19}$ C

$\frac{1}{4\pi\epsilon_0} = 9.0\times10^9\ \frac{Nm^2}{C^2}$

$r = 3.0\times10^{-15}$ m

$F_e = 9\times10^9\cdot\frac{(1.6\times10^{-19})^2}{(3\times10^{-15})^2} = 1.6^2\times10^9\cdot10^{-38}\cdot10^{30}$

$$\boxed{F_e \cong 2\times10^1 = \underline{20\ N}}$$ ~10^{36} x stronger than gravity

$F_{nuc} : F_e : F_g$

2000N : 20N : 2×10^{-35} N

$1 : 10^{-2} : 10^{-38}$

* Nuclear Force: recall that $F = -\frac{dU}{dx}$ ← potential energy or $F = \frac{\Delta U}{\Delta x}$

So $F_{nuc} = \frac{40\times10^6 eV\cdot1.6\times10^{-19} J/eV}{3\times10^{-15} m} = \frac{4\cdot1.6}{3}\times10^7\cdot10^{-19}\cdot10^{15} = 2\times10^3 = \boxed{2000\ N}$

~100X F_e

$\boxed{12\text{-}17}$ What is the energy released when 3 α particles combine to form ^{12}C?

$$3\ {}^{4}He \rightarrow {}^{12}C + Q$$

$$Q = 3\, M_{^4He}\, c^2 - M_{^{12}C}\, c^2$$

$$M_{^4He} = 4.002603\ u \qquad M_{^{12}C} = 12.0000$$

$$Q = (3 \cdot 4.002603 - 12.000) \times \underbrace{931.49\ \text{MeV}}_{\text{conversion from mass in atomic mass units to energy in eV.}}$$

$$= (12.007809 - 12.000) \times 931.49\ \text{MeV}$$

$$\boxed{Q = 7.27\ \text{MeV}}$$

$\boxed{12\text{-}24}$ (a) Use the semi-empirical mass formula to determine the binding energy per nucleon for ^{18}C, ^{18}N, ^{18}O, and ^{18}Ne

Use Eq. 12.20 $B = a_v A - a_A A^{2/3} - \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 r} - a_s \frac{(N-Z)^2}{A} + \delta$

where $a_v = 14\,\text{MeV}$, $a_A = 13\,\text{MeV}$, $a_s = 19\,\text{MeV}$ and

$$\delta = \begin{cases} +33\,\text{MeV}/A^{3/4} & \text{even-even} \\ 0 & \text{odd-even} \\ -33\,\text{MeV}/A^{3/4} & \text{odd-odd} \end{cases}$$

All the nuclei shown have $A = 18$, so the first two terms are the same...

$a_v A = 252\,\text{MeV}$ $\quad a_A A^{2/3} = 89.3\,\text{MeV}$

They all have the same radius... $r = r_0 A^{1/3} = 1.2\times10^{-15}\,\text{m} \cdot 18^{1/3} = \underline{3.14\times10^{-15}\,\text{m}}$

so the third term is... $-\frac{3}{5}\left(\frac{e^2}{4\pi\epsilon_0 r}\right)\cdot Z(Z-1) = 0.6\underbrace{\left[\frac{(1.6\times10^{-19}\,C)^2}{4\pi\epsilon_0 \cdot 3.14\times10^{-15}\,m}\right]}_{}Z(Z-1)$

units are Joules in mks, but convert to eV by dividing by e.

$$0.6\,\frac{1.6\times10^{-19}}{3.14\times10^{-15}}\cdot 9\times10^{9} = 2.75\times10^{5}\,\text{eV}$$

$\underline{\text{Third term}} = 0.275\,\text{MeV}\cdot(Z(Z-1))$

Now take each isotope individually:

$\boxed{^{18}C}$ $\quad A = 18,\ Z = 6,\ N = 12$ $\quad$ even-even

$$B = 252\,\text{MeV} - 89.3\,\text{MeV} - 0.275\,\text{MeV}(\overset{6.5}{30}) - 19\,\text{MeV}\,\frac{\overset{12-6}{6}^2}{18} + \frac{33\,\text{MeV}}{18^{3/4}}$$

$$= 252 - 89.3 - 8.25 - 38 + 3.78 = 120\,\text{MeV}$$

or.. $\boxed{\frac{B}{A} = 6.68\,\text{MeV}}$

$\boxed{^{18}N}$ $A=18$ $Z=7$ $N=11$ odd-odd

$$B = 252 - 89.3 - 0.275(42) - 19\cdot\frac{4^2}{18} - \frac{33}{18^{3/4}}$$

$$= 252 - 89.3 - 11.55 - 16.89 - 3.78 = 130 \text{ MeV}$$

$$\boxed{\frac{B}{A} = 7.25 \text{ MeV}}$$

$\boxed{^{18}O}$ $A=18$ $Z=8$ $N=10$ even-even

$$B = 252 - 89.3 - 0.275(56) - 19\,\frac{2^2}{18} + 3.78$$

$$= 252 - 89.3 - 15.4 - 4.22 + 3.78 = 147 \text{ MeV}$$

$$\boxed{\frac{B}{A} = 8.16 \text{ MeV}}$$

$\boxed{^{18}Ne}$ $A=18$ $Z=10$ $N=8$ even-even

$$B = 252 - 89.3 - 0.275(90) - 19\,\frac{2^2}{18} + 3.78$$

$$= 252 - 89.3 - 89.7 - 4.22 + 3.78 = 72.6 \text{ MeV}$$

$$\boxed{\frac{B}{A} = 4.03 \text{ MeV}}$$

(b) ^{18}O is the most stable. This is expected since it is an even-even nuclide with slightly more neutrons than protons. Empirically it is the only stable isotope among these

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*Note ^{18}F gives... $A=18$ $Z=9$, $N=9$ odd-odd

$$B = 252 - 89.3 - 19.8 - 0 - 3.78 = 139 \text{ MeV} \quad \boxed{\frac{B}{A} = 7.73 \text{ MeV}}$$

close to ^{18}O but still unstable according to Appendix 8

(c) Using atomic masses to calculate the binding energy of ^{18}O ....

$$B = Z M_H c^2 + N m_n c^2 - M_A c^2$$

$$= \left(8\left(1.007825\right) + 10\left(1.008665\right) - 17.999160\right) \times 931.49\ \text{MeV}$$

$$= \left(8.0626 + 10.08665 - 17.99160\right) \times 931.49\ \text{MeV}$$

$$= \left(0.15009\right) \times 931.49\ \text{MeV} = \underline{\underline{139.8\ \text{MeV}}}$$

This is close to the value of 147 MeV that the semi-empirical mass formula yields.

12-26 An unknown radioactive sample is observed to decrease in activity by a factor of five in a one hour time period. What is its half-life?

$$R = R_0 e^{-\lambda t} \quad \text{where} \quad t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

solve this for λ...

$$\frac{R_0}{R} = e^{\lambda t} \quad \text{so...} \quad \lambda t = \ln\left(\frac{R_0}{R}\right)$$

or..

$$\lambda = \frac{1}{t} \ln(R_0/R).$$

so..

$$\boxed{t_{\frac{1}{2}} = \frac{t \ln 2}{\ln(R_0/R)}}$$

For $t = 60$ min and $R_0/R = 5$

$$t_{\frac{1}{2}} = \frac{60\text{min} \cdot 0.693}{\ln 5} = \boxed{25.8 \text{ minutes}}$$

This makes sense since 60 min is a little more than 2 half-lives, so..

$$R \lesssim \frac{R_0}{2^2}$$

12-27 Show that the mean (or average) lifetime of a radioactive sample is $\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$.

$$\text{Avg. lifetime} = \frac{\sum_i (\#\text{ that live } t_i) \times t_i}{\text{total \# of particles}}$$

$$\text{The } (\#\text{ that live } t_i) = \underbrace{(\text{decay rate at time } t_i)}_{R(t_i)} \times \Delta t$$

↑ small time interval

Sum becomes an integral

$$\tau = \frac{1}{N_0} \int_0^\infty R(t)\, t\, dt$$

where $R(t) = \lambda N(t) = \lambda N_0 e^{-\lambda t}$

So...

$$\tau = \frac{1}{N_0} \int_0^\infty \lambda N_0 t e^{-\lambda t} dt = \lambda \underbrace{\int_0^\infty t e^{-\lambda t} dt}_{\text{use integration by parts...}}$$

$\int u\,dv = uv - \int v\,du$ with $u = t$, $dv = e^{-\lambda t} dt$

$du = dt$, $v = -\frac{1}{\lambda} e^{-\lambda t}$

Then..

$$\tau = \lambda \left[\underbrace{-\frac{t}{\lambda} e^{-\lambda t} \Big|_0^\infty}_{\text{goes to zero at both limits.}} + \int_0^\infty \frac{1}{\lambda} e^{-\lambda t} dt \right]$$

$$\tau = \int_0^\infty e^{-\lambda t} dt = -\frac{1}{\lambda} e^{-\lambda t} \Big|_0^\infty = 0 - \left(-\frac{1}{\lambda}\right) = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$$

QED

12-30 The half-life of ^{18}F is 109.8 min. If the initial activity is 1.0×10^7 Bq, what is the activity 48 hours later? This is more than 25 half-lives!

First find the decay constant λ : $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{109.8\,min} = 0.00631\ min^{-1}$

The activity decays according to the equation : $R = R_0 e^{-\lambda t}$

So, let $R_0 = 1.0 \times 10^7$ Bq , $\lambda = 0.00631\ min^{-1}$, and $t = 48\ hrs. \times 60\ ^{min}/_{hr} = 2880\,min$

$$R = 1.0 \times 10^7\,Bq \cdot e^{-0.00631 \cdot 2880} = 1.0 \times 10^7\,Bq \cdot e^{-18.17} = 0.13\,Bq$$

activity has reduced by more than 8 orders of magnitude.

$\boxed{12\text{-}37}$ How much kinetic energy does the daughter have when ^{241}Am undergoes α decay from rest?

First calculate the decay energy for this decay..

$$^{241}Am \rightarrow {}^{237}Np + {}^{4}He$$

$$Q = M_P c^2 - M_D c^2 - M_{He} c^2$$

$$= (241.056823\,u - 237.048167\,u - 4.002603\,u) \times 931.49\,\frac{MeV}{uc^2}$$

$$= 0.00605\,u \times 931.49\,\frac{MeV}{u} = 5.64\text{ MeV}$$

The alpha particle carries away most of this energy. Using conservation of momentum one can derive Eq. 12.32..

Alpha particle kinetic energy $K_\alpha \approx \left(1 - \frac{4}{A}\right) Q = \left(1 - \frac{4}{241}\right) Q$

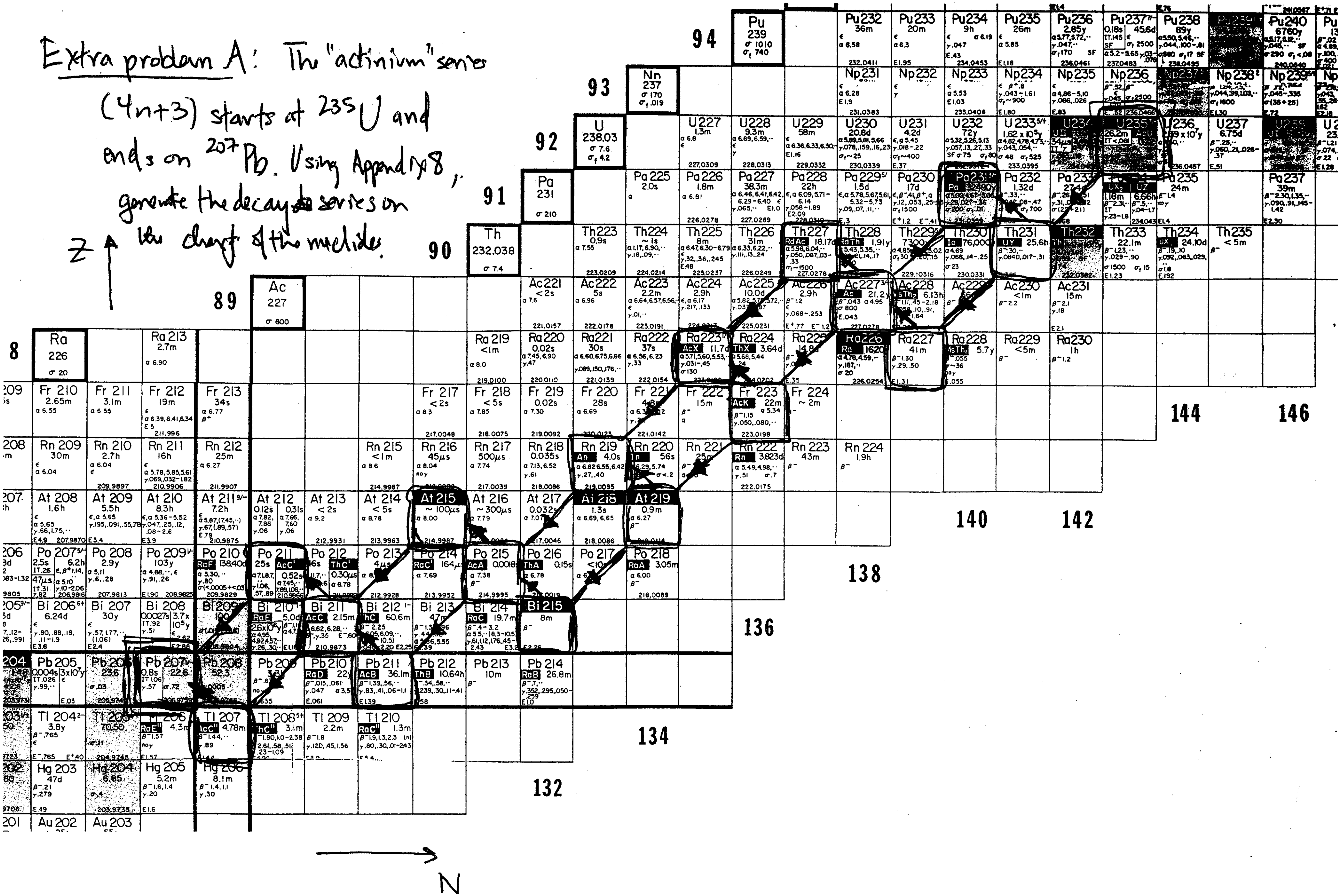
$$= 0.938 \cdot Q = 5.55\text{ MeV}$$

The daughter nucleus carries away the remainder of the decay energy...

$$K_D = Q - K_\alpha = \frac{4}{A} Q = 0.0166\, Q = 0.094\text{ MeV}$$

$$= \boxed{94\text{ keV}}$$

Extra problem A: The "actinium" series ($4n+3$) starts at ^{235}U and ends on ^{207}Pb. Using Appendix 8, generate the decay series on the chart of the nuclides.



Extra Problem B: Write down the decay reactions and calculate the decay energy.

(i) α decay of ^{226}Ra :

$$\boxed{{}^{226}Ra \rightarrow {}^{222}Rn + {}^{4}He} \quad t_{\frac{1}{2}} = 1600y$$

$$Q_\alpha = M_P c^2 - M_D c^2 - M_{He} c^2$$

$$= (226.025403u - 222.017570u - 4.002603\,u) \times 931.49 \frac{MeV}{uc^2} \cdot c^2$$

$$= 0.00523u \times 931.49\, MeV/u = \boxed{4.87\, MeV}$$

(ii) β^- decay of ^{40}K

$$\boxed{{}^{40}K \rightarrow {}^{40}Ca + e^- + \bar{\nu}_e} \quad t_{\frac{1}{2}} = 1.28 \times 10^9 y$$

$$Q_{\beta^-} = M_P c^2 - M_D c^2 = (39.963999u - 39.962591u) \times 931.49 \frac{MeV}{u}$$

$$= 0.001408u \times 931.49 \frac{MeV}{u} = \boxed{1.31\, MeV}$$

(iii) β^+ decay of ^{22}Na

$$\boxed{{}^{22}Na \rightarrow {}^{22}Ne + e^+ + \nu_e} \quad t_{\frac{1}{2}} = 2.60\, yr$$

$$Q_{\beta^-} = M_P c^2 - M_D c^2 - 2m_e c^2$$

$$= (21.994437u - 21.991386u) \times 931.49 \frac{MeV}{u} - 2(0.511\, MeV)$$

$$= 0.003051 \times 931.49\, MeV - 1.022\, MeV =$$

$$= 2.842\, MeV - 1.022\, MeV = \boxed{1.82\, MeV}$$

(iv) electron capture decay of ^{57}Co :

$$\boxed{^{57}Co + e^- \rightarrow {}^{57}Fe + \nu_e} \quad t_{\frac{1}{2}} = 272 \text{ days}$$

$$Q_\epsilon = M_p c^2 - M_D c^2$$

$$= (56.936296u - 56.935399u) \times 931.49 \frac{MeV}{u}$$

$$= 0.000897u \times 931.49 \, MeV/u = \boxed{0.836 \, MeV}$$