Math 300: Foundations of Algebra

Winter Term, 2010, Lawrence University

Professor: Scott Corry
Office: 408 Briggs Hall, x7287
Office Hours: MTWR 3-4:15, and by appointment
E-mail: scott.corry@lawrence.edu
Webpage: www.lawrence.edu/fast/corrys
Text: C.C. Pinter, *A Book of Abstract Algebra*, 2ed.

1 Overview

This course has two goals, the first of which is to learn some abstract algebra, in particular elementary group theory through the Sylow Theorems and a bit of ring theory. This material will be unfamiliar to you, and you will find it challenging. However, you will find our second goal even more challenging: to change the way you think about mathematics. Probably for the first time in your lives, you will be asked to interact with mathematical objects in a serious, formal, and abstract way. You will be forced to give up on the idea of knowing *what something is* in favor of knowing *how it behaves*.

The previous sentence probably makes little sense to you, but actually you are already used to working with objects without knowing what they are, only how they behave: think of the real numbers. Each of you would be hard pressed (for good reason) to tell me what a real number *is*, yet you have no problem working with them. You add and subtract them, multiply and divide them, and you know all sorts of facts about these arithmetic operations, for example the distributive law. Of course, the real numbers are very special, and the *way* in which they are special makes them the right setting for the differential and integral calculus. But that is a story for another day and another course (Math 310, *Foundations of Analysis*). In *this* course we move in the opposite direction, and instead of lamenting our lack of knowledge of

the real numbers, we celebrate it, and ask ourselves questions such as: in what sorts of settings does it make sense to do things like add, subtract, multiply, and divide? What are the consequences of the distributive law?

If you are used to thinking of algebra as the manipulation of real numbers subject to familiar rules, then *abstract* algebra is the study of rules and their logical consequences, without regard for the *meaning* of the symbols being manipulated.

The bulk of our course will be devoted to *groups*, which are sets with only *one* operation, satisying a few axioms. Towards the end of the course we will study *rings*, which are sets with *two* operations, satisfying several axioms. Groups and rings are the most important *algebraic structures*, and it will be our business to study them in depth (especially groups), invite them into our lives, and ask personal questions about them: What types of groups are there? When are two groups the same? How are two different groups related? Can we describe in some reasonable way what groups look like? How can we put two groups together to make another group? Are there special groups that serve as fundamental building blocks out of which all other groups are made?

2 Coursework

Work for this course will come in four types:

1. *Preparation and Study*: You must read the assigned sections of the text BEFORE I lecture on them, so that you are already working with the ideas in advance of hearing about them from me. I cannot emphasize enough how helpful this simple practice is. In addition, review your lecture notes after each lecture, carefully reconstructing *for yourself* the ideas, arguments, and overall algebraic story that is developing. Listening to someone else talk for 70 minutes 3 times a week is simply not sufficient to learn mathematics and reorganize your thought processes.

2. *Basic Problems*: Before (and during) every lecture I will assign a few basic problems for you to work on before the next lecture. You will not hand these in to be graded, but you should do them as a way of keeping yourself engaged and checking your progress. You should think of these as the diagnostic exams that a doctor might perform: if the result comes back negative (you **can** do the problems), then you

appear to be OK, although you might still be sick. On the other hand, if the result comes back positive (you **can't** do the problems), then you are definitely sick, and need immediate treatment. The treatment is to come talk to me during office hours, talk to each other, study harder, review your notes and reread the text, etc. Don't neglect your treatment . . . untreated illnesses tend to fester.

3. *Challenging Problems*: You will have 6 challenging problem sets, due roughly every week and a half (schedule below). These will require serious thought and a substantial time commitment. In particular, if you wait until a couple of days before they are due, you are unlikely to have success. You should start working on the problems immediately, and work on them a little (or a lot) every day. You must live and sleep with these concepts and problems, and often let them run in the background while your conscious mind is doing other things. Only by this deep and long immersion will you become accustomed to abstraction and the type of argumentation that is essential for serious mathematics. These problems are supposed to be difficult and frustrating, and there will be problems that elude even your best efforts. Because it is the process much more than the result that is important here, I will be reluctant to help you in office hours on the problem sets: it is much better for you to knock your head against something for a week or so with only partial success, than to get the answer from me after a couple of hours or days of work.

Schedule of Due Dates for Problem Sets

- Problem Set 1: due Wednesday, January 13
- Problem Set 2: due Friday, January 22
- Problem Set 3: due Wednesday, February 3
- Problem Set 4: due Wednesday, February 10
- Problem Set 5: due Monday, March 1
- Problem Set 6: due Wednesday, March 10

Problem sets are due at the beginning of class and NO LATE WORK WILL BE ACCEPTED (except under extraordinary circumstances).

4. *Exams*: There will be one in-class midterm exam (Monday, February 15) and a take-home final exam. The final exam will be distributed on the last day of classes (Friday, March 12), and will be due by 4:30 p.m. on Friday, March 19.

3 Evaluation

Your course grade will be determined as follows:

- *Problem Sets* 60%
- *Midterm Exam* 15%
- *Final Exam* 25%

In grading the challenging problem sets, half of the total points will be awarded based solely on evidence of a sustained and substantial effort. This means that if you turn in a serious and well-written solution to every problem, then you are guaranteed at least half credit on your assignment. The other half of the points will be awarded based on the correctness of your arguments. I have chosen this rather unorthodox system in order to underscore the point that it is the *process* of thinking and writing that we are primarily concerned with in this course. Hence, even if you are unable to successfully complete a problem, I expect you to write a careful account of your attempt, your ideas, and the pitfalls you encountered. I think you will find that the process of writing and explaining is often instrumental in actually constructing a solution. In fact, learning to write mathematics carefully and correctly is an auxiliary goal of this course. This can only be accomplished through practice, so for this course, you MUST write up your problem sets alone, with no help from others. I encourage you to work together to generate ideas and arguments, but I expect you to walk away from these group sessions with nothing more than scratch paper and a better understanding. You should expect to spend a substantial amount of time writing down a *polished* and *carefully crafted* account of your work. By affirming the Lawrence University Honor Code on your paper, you will be testifying that your final exposition is entirely original and the product of a sustained period of isolated writing.