Math 545: Rings and Fields

Winter Term 2009, Lawrence University

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Text: J. Swallow, *Exploratory Galois Theory* (Cambridge)

1 Overview

This is a second course in abstract algebra, and our goal is to develop Galois Theory for subfields of the complex numbers. After a brief review of rings in general, we will restrict our attention to polynomial rings. In so doing we will appreciate a striking analogy between polynomials and integers. In investigating this analogy, we will discover the importance of irreducible polynomials (the analogues of prime numbers), and we will develop some tests for irreducibility.

The notion of an algebraic number will motivate our discussion of field extensions and the isomorphisms between them. We will eventually introduce the Galois group of a field extension, and describe a beautiful correspondence between the structure of a field extension and the structure of its Galois group. Hence, by the end of the course, we will be employing the group theory that you learned in *Math 300: Foundations of Algebra* in order to understand the algebraic numbers and fields that are the subject matter of this course. Finally, we will see some classical applications of Galois Theory to Euclidean geometry and to the solvability of polynomial equations by radicals.

Many of the assignments for this course will involve using the computer algebra package *Mathematica* to compute nontrivial examples. No prior experience with *Mathematica* is assumed. In fact, our text includes a sort of guided tutorial for using *Mathematica* to explore the concepts of Galois Theory. You each have access to *Mathematica* on the machines in Briggs 419.

2 Practicalities

2.1 Homework

There will be regular homework assignments, and selected problems will be carefully graded on the following five point scale:

5 = perfect – correct and well-written
4 = one minor error
3 = one major error or several minor errors
2 = several major errors
1 = indicative of relevant thought

Remember that the quality and clarity of your writing are important. Take the time to make your solution set neat and legible, and strive to effectively convey your ideas. You may each make use of **one** homework extension until the class period after the initial due date of an assignment. Homework will count for 70% of your final grade, and except for your one-time extension, **absolutely no late homework will be accepted**.

2.2 Exams

We will have a take-home midterm as well as a take-home final exam. Each will count for 15% of your final grade.

2.3 Office Hours and Other Details

Please feel free to come by my office, whether you are having difficulty or just want to chat about mathematics. Of course, I may not be in, or I may be otherwise engaged and unable to talk. To ensure that you have my undivided attention, you should come during my office hours, which are listed above. Of course, if these times are impossible for you, you can always make an appointment with me.

In addition to talking with me, I encourage you to speak with each other about the course, and even to work together on the problems if that suits your style of learning. That being said, I expect you to spend some time thinking privately about the problems before collaborating, and each of your writeups should be the result of your own cogitation and exposition. If you like to work together, a good model would be to make a first pass through the problems on your own, then get together with friends to talk about difficulties and share ideas, and finally find a solitary place to write a polished (and unique) solution set. Of course, all of your work for this course is governed by the Lawrence University Honor Code.