

Answers to Problem Set #5

1. There are two ways one can proceed: attempt to solve directly for  $Y_d$  or first solve for  $Y$  and then solve for  $Y_d$ . I prefer the second since all other endogenous variables can be derived easily from the reduced form equation for  $Y$ .

Plug (4) into (3) and the result into both (1) and (2)

$$Y_d = Y - 50 - .1*Y = .9*Y - 50 \quad (3')$$

$$C = 200 + .8* [.9*Y - 50] = 200 + .72*Y - 40 = 160 + .72*Y \quad (1')$$

$$I_m = 25 + .2* [.9*Y - 50] = 25 + .18*Y - 10 = 15 + .18*Y \quad (2')$$

Now plug (1') and (2') into (5) and set (5) = (6)

$$Y = 160 + .72*Y + I_p + G + eX - 15 - .18*Y = 145 + .54*Y + I_p + G + eX$$

Now solve for  $Y$

$$Y - .54*Y = .46*Y = 145 + I_p + G + eX \text{ which implies that}$$

$$Y = \frac{145 + I_p + G + eX}{.46}; \text{ the reduced form for } Y_d = .9*Y - 50 = \frac{145 + I_p + G + eX}{.511} - 50$$

Given the values indicated in the problem  $Y = 960/.46 = 2087$

from (4)  $Y_d = .9*Y - 50 = 1828.3$

- b. From (1')  $C = 160 + .72*2087 = 1662.6$  and (2')  $I_m = 15 + .18*2087 = 390.7$   
Since  $S = Y - C - T - I_m = Y_d - C - I_m = 1828.3 - 1662.6 - 390.7 = -225.0$
- c. The equilibrium level of taxes is the difference between  $Y$  and  $Y_d = 2087 - 1828.3 = 258.7$   
The government deficit  $G - T = 91.3$
- d. Based on the reduced form statement for  $Y$ , a \$100 change in  $G$  yields a  $100/.46$  change in  $Y = 217$ . This can be checked by replacing  $G = 350$  with  $G = 450$ .
- e. Replace  $t_1 = .1$  in equation (4) with  $t_1 = .2$ . As a result,  $Y_d = .8*Y - 50$ ;  
 $C = 160 + .64*Y$ ; and  $I_m = 15 + .16*Y$ .  
After using equations (5) and (6), we find  $Y = 960/.52 = 1846$   
Thus, a .1 change in  $t_1$  yielded a 241 point change in the opposite direction in  $Y$ .  
Taxes would be given by:  $T = 50 + .2*1846 = 419.2$ ; Now the government would run a surplus of 69.2 ( $T - G$ ).

2. a. To equations (1) - (3) we need to add  
(4)  $Y_d = Y - T$   
(5)  $Ad = C + I + G + eX - I_m$   
(6)  $Y = AD$
- b. To find the reduced form, plug (4) into (1) and (3) and the results into (5), which is set equal to (6). Solve for  $Y$ .  
(1)  $C = 5000 + .9*Y_d = 5000 + .9*Y - .9*[-1000 + .33Y] = 5900 + .6*Y \quad (1')$   
(3)  $I_m = 500 + .15*Y_d = 500 + .15*Y - .15*[-1000 + .33*Y] = 650 + .1*Y \quad (2')$   
(5), (6)  $Y = C + I_p + G + eX - I_m = 5900 + .6*Y + I_p + G + eX - 650 - .1*Y$   
 $Y - .5*Y = .5*Y = 5250 + I_p + G + eX$ ; thus,  $Y = \frac{5250 + I_p + G + eX}{.5}$

- c.  $Im = 500 + .15*Y_d = 650 + .1*Y = 1700 + .2*[I_p + G + eX]$
- d. A look at equation (5) suggests that the exogenous part of imports and governmental expenditures enter AD in the same way; thus, G would have to rise by 500 to match the rise in imports.

3a. I will provide the answer to part a. The answer to b. follows the result derived in class. Consider M endogenous and r exogenous. Interpretation: The Federal Reserve targets an interest rate and lets the stock of money adjust to whatever is needed to reach the desired level.

Strategy: Solve (1) - (6) for the IS curve.

First, plug (3) into (2) and then (2) into (1)

$$Y_d = Y - .25*Y = .75*Y$$

$$C = a + .8* [.75*Y] = a + .6*Y \quad (1')$$

Now plug (1') and (4) into (5) and set (5) = (6)

$$Y = AD = a + .6*Y + e - 10*r + G$$

$$\text{which can be solved for } Y = \frac{a + e - 10*r + G}{.4} \quad (6')$$

Since r is exogenous, (6') serves as both the IS curve and the reduced form statement for Y. Set (7) = (8) and solve for M in terms of r since r is exogenous.

$M = (.25*Y - 5*r)*P$  Since Y is given by the IS curve in this case and since r and P are exogenous, this LM curve is the reduced form equation for M.

c. In part a, monetary policy consists of changes in r. For part a, a 1 percentage point change in r yields a 25 unit (10/.4) change in Y in the opposite direction. In part b, it consists of changes in M; i.e., the multiplier for changes in real balances on income is  $2/.9 = 2.22$ . Given  $P = 2$ , changes in M have an income multiplier of 1.11.

d. Fiscal policy can be analyzed through either changes in G or t. Given the model in part a, for G, the multiplier is  $[1/.4] = 2.5$ ; thus, for each dollar change in G, Y changes by 2.5 dollars in the same direction. For part b, the multiplier is  $1/.9 = 1.11$ .

e. The reduced form equation for Y, derived by plugging the LM curve value for r into the IS curve is

$$Y = \frac{a + e + G - 10*[-.2*M/P]}{1 - .6 + 10*(.25/5)} = (100 + 200 + 300 + 300)/.9$$

$$\text{Thus, equilibrium } Y = 1000$$

To reach natural Y ( $Y^* = 1200$ ), we must find the value of M that increases Y by 200.

The multiplier of money on output is given by  $-10*(-.2/P)/.9 = (2/P)/.9 = 1.11$ ; thus, we would need to increase M by  $200/1.11 = 180$  to bring Y up to 1200.

f. As in e, we want to increase Y by 200, the governmental expenditures multiplier =  $1/.9 = 1.11$ , the same as the money multiplier; thus, to reach  $Y^*$ , governmental expenditures should be increased by 180.

4. To be able to derive the IS curve, we need to add the following equations:

$$(6) AD = C + I + G + eX - Im$$

$$(7) Y_d = Y - T$$

$$(8) Y = AD$$

$$(9) M_d = M \text{ with } M \text{ exogenous}$$

Now plug (1), (2), and (4) into (6).

$$Y = 700 + .90*Y_d + 500 - 40*r + G + eX - 200 - .15*Y_d$$

which can be collapsed into

$$(6') Y = 1000 + .75*Y_d - 40*r + G + eX$$

Now plug (3) into (7) to yield:  $Y_d = Y - .33*Y = .67*Y$  and the result into (6')

$$Y = 1000 + .75*(.67)*Y - 40*r + G + eX \text{ which can be solved for } Y$$

$$\text{IS: } Y = \frac{1000 - 40*r + G + eX}{.5} \text{ or } Y = 2000 - 80*r + 2*G + 2*eX$$

with slope  $\Delta r / \Delta Y = -1/80$ .

b. Now use equation (5) along with the money market equilibrium condition (9) and solve for  $r$  to describe the LM curve.

$$M_d = M \text{ and } M_d/P = .25*Y - 10*r$$

thus,  $r = -(1/10)*M/P + (.25/10)*Y$  gives the LM curve with slope  $\Delta r / \Delta Y = .025$

c. Now plug the LM curve into  $r$  in the IS curve.

$$Y = 2000 - 80*[-(1/10)*M/P + (.25/10)*Y] + 2*G + 2*eX$$

Multiply out the terms in brackets and then solve for  $Y$

$$Y = 2000 + 8*(M/P) - 2*Y + 2*G + 2*eX$$

and finally

$$Y^* = 2000/3 + (8/3)*(M/P) + (2/3)*G + (2/3)*eX$$

d. The money multiplier is given by the second term on the right hand side.

For  $P = 2$ ,  $\Delta Y / \Delta M = 4/3$ .

e. The fiscal policy multiplier is given by the coefficient of  $G$  in the reduced form equation:

$$\Delta Y / \Delta G = 2/3$$

f. Since the money multiplier is larger, monetary policy is more potent than fiscal policy.