Macroeconomic Theory
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Answers to Problem Set 4

1a. Begin with $Y = 100*K^{.3}L^{.7}$ and solve for the per worker form by dividing by L to yield
$y = Y/L = 100*K^{.3}L^{.7}/L = 100 * (K/L)^{.3}$ which $= 100*k^{.3}$

b. Using Model 1G and substituting in the specifics given in this problem (n=0), the steady state values for y and k are as follows:
$y = 100^{1.43}*[s/\ast]^{.43}$ $k = 100^{1.43}*[s/\ast]^{1.43}$ and $c = (1-s)* 100^{1.43}*[s/\ast]^{.43}$

c. max y (by choosing s) $= \ast y/\ast s = -100^{1.43}*[s/\ast]^{.43} + (1-s)*(.43/\ast)*100^{1.43}*[s/\ast]^{-.57} = 0$
When we solved for s, we find that it equals 0.3.

d. The share of labor = w*L/Y = MPL*L/Y in a competitive economy.
The share of capital = rent*K/Y = MPK*K/Y
$MPL = .7*100* K^{.3}L^{-.3}$ -- so labor's share $= .7*100* K^{.3}L^{-.3}*L/Y = .7$
$MPK = .3*100*K^{-.7}L^{.7}$ -- so capital's share $= .3*100*K^{-.7}L^{.7}*K/Y = .3$
Labor and Capital's share sum to 1.

2a. Here $y = k^{.5}*L^{.2}$ -- so y cannot be written solely in terms of k.

b. As above, we plug the values in and determine that
Long run demand is given by $y = (\ast/s)*k$. Using this equation with the production function in 4a yields: $k = L^{.04} * (s/\ast)^2$ and $y = L^{.04} * (s/\ast)$ with $c = (1-s)*(s/\ast) L^{.04}$

c The optimal savings rate comes from maximizing c with respect to s. Taking the partial derivative and setting it to zero yields s = .5

d. The share of labor = w*L/Y = MPL*L/Y in a competitive economy.
The share of capital = rent*K/Y = MPK*K/Y
$MPL = .7*K^{.5}L^{-.3}$ -- so labor's share $= .7* K^{.5}L^{-.3}*L/Y = .7$
$MPK = .5*K^{-.5}L^{.7}$ -- so capital's share $= .5*K^{-.5}L^{.7}*K/Y = .5$

Labor and Capital's share sum to 1.2 or twenty percent greater than the amount of output generated. This is not feasible so competition in both factor markets cannot hold.

3. Technological progress can be introduced in three ways in the Solow Model

   i) It can be independent of either capital or labor
   ii) It can be embedded in the purchase of capital
   iii) It can be tied to labor through growth in human capital

a. For the first option, growth in technology goes directly into growth in output 1 for 1. Growth in capital enters with a weight less than one to reflect diminishing returns ($b < 1$)

b. Given the Cobb-Douglas production function, effect of technological improvement depends on the exponent. For the basic model $Y = Tech*K^{b}L^{1-b}$, growth is determined by

$$Y\ gr = Tech\ gr + b*Kgr + (1-b)*L\ gr.$$

For $0<0.5<b$, the influence is in the order i,iii, ii.
For $0.5 < b < 1$, the order of influence would be i, ii, iii.

4a. Divide both sides of the production function by L to yield
$Y/L = (K)^{1/3}\ L^{-1/3}$ which can be rewritten as $\mathbf{y = k^{1/3}}$ **(A) – long run supply.**
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b. We know from the model (equation 6) that capital growth must equal labor growth along the long run path.. But we are given from the question that K grows at 2%, call it g; therefore, labor must also grow at g. Since capital growth is given by $s*(y/k) – \delta = g$, we can solve for y to yield $\mathbf{y = (g+ \delta)*k/s}$ **(B) - long run demand**

Now, we put our two equations (A and B) together to yield $\mathbf{y = (s/(g+ \delta))^{.5}}$ **(C)**
In this case, that equals $(.1/(.02+.1)^{.5} = .913$

c. Since $y = (s/(g+ \delta))^{.5}$ and $k = y^{3}$, $k = (s/(g+ \delta))^{1.5}$
so the capital/ output ratio = $(s/(g+\delta))$ – is not a function of time

d. From the production function, we find that output per laborer will grow at the following $(1/3)*Kgr + (2/3)*Lgr - Lgr = (1/3)*(.02) + (2/3)*(.02) - .02 = 0\%$. Output, however, will grow at 2.0%, the same rate as capital and labor.

e. Changes in the depreciation rate affect the denominator in C. If $*$ doubles, y must fall. Since output = $y*L$ and L is not a function of δ, it would fall by a similar percentage.