

Answers to Problem Set #8

Chapter 15

1. The five equations that make up the dynamic aggregate demand–aggregate supply model can be manipulated to derive long-run values for the variables. In this problem, it is assumed that there are no shocks to demand or supply and inflation has stabilized. Since inflation has stabilized, it must be true that inflation in time t is equal to inflation in time $t - 1$ ($\pi_t = \pi_{t-1}$). We also know that expected inflation is equal to last period's inflation, or $E_{t-1}\pi_t = \pi_{t-1}$. Start with the Phillips curve equation below and use these two facts to find the following:

$$\begin{aligned}\pi_t &= E_{t-1}\pi_t + \Phi(Y_t - Y_t\text{bar}) + v_t \\ \pi_t &= \pi_{t-1} + \Phi(Y_t - Y_t\text{bar}) + v_t \\ \pi_t &= \pi_t + \Phi(Y_t - Y_t\text{bar})\end{aligned}$$

From here, it follows that output must equal natural output. Moving to the demand for goods and services equation next, it now follows that the real interest rate equals the natural rate of interest since the demand shock parameter ε_t equals zero and $Y_t = Y_t\text{bar}$:

Turning to the Fisher equation, we can show the nominal interest rate is equal to the natural interest rate plus the current inflation rate. Since inflation has stabilized, expected inflation equals current inflation ($E_t\pi_{t+1} = \pi_t$) and we have just demonstrated that the real interest rate is equal to the natural rate of interest ($r_t = \rho$):

Moving now to the monetary policy rule equation, it must be true that current inflation equals the target inflation rate so that the third term on the right zeros out. Likewise, the fourth term on the right side will zero out since output is at the natural level:

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(Y_t - Y_t\text{bar})$$

3. “If a central bank wants to achieve lower nominal interest rates, it has to raise the nominal interest rate.” In long-run equilibrium, the nominal rate of interest is equal to the natural rate of interest plus the target inflation rate. To lower the long-run nominal interest rate, the central bank must lower the target inflation rate and, ultimately, the actual inflation rate. In the short run, the central bank must increase the nominal interest rate in order to reduce spending and output in the economy. This will reduce inflation and, ultimately, expected inflation. The economy will adjust to a new long-run equilibrium in which the nominal interest rate, the target inflation rate, and the actual inflation rate are all lower. Graphically, lowering the target inflation rate will shift the dynamic aggregate demand curve down and to the left, forcing the economy through a recessionary cycle, and in the short run, output and inflation will be lower. As

expected inflation adjusts over the long run, the dynamic aggregate supply curve will shift down and to the right. In the long run, output is equal to the natural level and inflation is lower.

5. Follow the hint given in the problem and solve for the long-run equilibrium with the new assumption that the demand shock parameter ε_t is not zero. Since inflation has stabilized, it must be true that inflation in time t is equal to inflation in time $t - 1$ ($\pi_t = \pi_{t-1}$). We also know that expected inflation is equal to last period's inflation, or $E_{t-1}\pi_t = \pi_{t-1}$. Start with the Phillips curve below and use these two facts to find the following:

$$\pi_t = E_{t-1}\pi_t + \Phi(Y_t - Y_t\text{bar}) + v_t$$

$$\pi_t = \pi_{t-1} + \Phi(Y_t - Y_t\text{bar}) + v_t$$

$$\pi_t = \pi_t + \Phi(Y_t - Y_t\text{bar})$$

From here, it follows that output must equal natural output since the supply shock parameter v_t equals zero. From the demand for goods and services equation below, it now follows that the real interest rate equals the natural rate of interest plus a new term:

$$Y_t = Y_t\text{bar} - \alpha(r_t - \rho) + \varepsilon_t \text{ which can be solved for } r_t = \rho + \varepsilon_t / \alpha$$

Turning to the Fisher equation below, we can show the nominal interest rate is equal to the natural interest rate plus the current inflation rate plus a new term. Since inflation has stabilized, expected inflation equals current inflation ($E_t\pi_{t+1} = \pi_t$), and we have just demonstrated that the real interest rate is equal to the natural rate of interest plus a term based on the error term in the IS curve:

$$r_t = \rho + \varepsilon_t / \alpha \text{ and } r_t = i_t - E_t\pi_{t+1} = i_t - \pi_t \text{ so } i_t = \rho + \varepsilon_t / \alpha + \pi_t$$

Moving now to the monetary policy rule equation below, substitute in for the nominal rate of interest from the rewritten Fisher equation above, and then note that the fourth term on the right side will zero out since output is at the natural level:

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(Y_t - Y_t\text{bar}) = \rho + \varepsilon_t / \alpha + \pi_t \text{ which can be manipulated to yield}$$

$$\pi_t = \pi_t^* + \varepsilon_t / \theta_\pi \alpha$$

If the demand shock parameter ε_t were to increase permanently, such that it remained a constant positive number, the dynamic aggregate demand curve would shift to the right permanently. This would cause a short-run increase in output and inflation and a long-run increase in the inflation rate as the economy adjusted to its new long run equilibrium. This is consistent with the newly derived long-run values above, where the inflation rate is higher than the target inflation rate. Note that expected inflation is also higher, as is the nominal and the real interest rate. To deal with this issue, the central bank could decrease its target inflation rate. This would effectively offset the permanent increase in the demand shock parameter ε_t and shift the dynamic aggregate demand curve back to its original position.

7. Suppose that the natural rate of interest is not a constant parameter but varies over time so that it is now written as ρ_t .

a. The equation for dynamic aggregate supply is not affected by this change because its derivation does not involve the natural rate of interest. The equation for dynamic aggregate demand is not affected by this change either because, although the variable ρ_t is involved in the derivation of the dynamic aggregate demand curve, it cancels out and does not end up as part of the final equation.

b. A shock to ρ_t would not cause a shift to either dynamic aggregate demand or dynamic aggregate supply because the variable does not appear in either equation. Output and inflation would not be affected. However, the real and nominal interest rates would both change by the amount of the change in ρ_t .

c. If the natural rate of interest varied over time, it would make the setting of monetary policy more difficult. If the central bank knows that the natural rate of interest is 4 percent, for example, and it is aiming for target inflation rate of 2 percent, then a nominal interest rate of 6 percent will be the long-run target. If, on the other hand, the natural rate of interest varies over time, then the target long-run interest rate will also vary over time. It is more difficult to hit a moving target than a target that is standing still. In particular, in contrast to what is implicitly assumed above, if the natural rate of interest is always moving, the central bank might have trouble knowing the natural rate of interest at every point in time.

9. Use the dynamic *AD–AS* model to solve for inflation as a function of only lagged inflation and the two shocks. Start with the dynamic aggregate supply curve and substitute in for Y_t using the dynamic aggregate demand curve equation as is done below. Now, solve for inflation through a few algebraic manipulations:

$$\pi_t = \pi_{t-1} + \Phi[Y_t - [\alpha\theta_\pi/(1 + \alpha\theta_y)](\pi_t - \pi_t^*) + [1/(1 + \alpha\theta_y)] \varepsilon_t - \bar{Y}_t] + v_t$$

Bring all π_t terms to the left hand side and divide through by its coefficient to yield

$$\pi_t = (1 + \alpha\theta_y) \pi_{t-1} / (1 + \alpha\theta_y + \Phi \alpha\theta_\pi) + \Phi \alpha\theta_\pi \pi_t^* / (1 + \alpha\theta_y + \Phi \alpha\theta_\pi) + \Phi \varepsilon_t / (1 + \alpha\theta_y + \Phi \alpha\theta_\pi) + (1 + \alpha\theta_y) v_t / (1 + \alpha\theta_y + \Phi \alpha\theta_\pi)$$

a. A supply or demand shock will lead to an increase in current inflation. As the economy adjusts and returns to long-run equilibrium, the inflation rate will return to its target level. Note that the coefficient on the lagged inflation variable in the equation above is positive but < 1 . This means that inflation in time $t + 1$ will be less than inflation in time t , and that inflation will eventually return to its target rate.

b. If the central bank does not respond to changes in output so that θ_y is zero, then the economy will still return to its target inflation rate after a supply or demand shock because the coefficient on the lagged inflation variable in the equation above is still positive but less than 1.

In this case, inflation should return more quickly to its target rate. This is because the coefficient on lagged inflation has become smaller (the change in the numerator is larger in comparison to the change in the denominator). The dynamic aggregate demand curve is relatively flat when the central bank only cares about inflation.

c. If the central bank does not respond to changes in inflation so that θ_π is zero, then the coefficient on lagged inflation in the above inflation equation equals 1. In this case, the economy will not return to its target inflation rate after a demand or supply shock. The demand or supply shock will increase inflation in time t . When θ_π is zero, inflation in time $t + 1$ is equal to inflation in time t .

d. The Taylor rule says that a one-percentage-point increase in inflation will increase the nominal interest rate by $1 + \theta_\pi$ percentage points. If the central bank increases the nominal interest rate by only 0.8 percentage points for each one-percentage-point increase in the nominal interest rate, then this means θ_π is equal to -0.2 . When θ_π is negative, the dynamic aggregate demand curve is upward sloping. A shock to demand or supply will set the economy on a path of ever-increasing inflation. This path of ever-increasing inflation will occur because real interest rates will continue to fall and output will remain above the natural level. You can see this phenomenon in the above equation for inflation: If θ_π is negative, the coefficient on lagged inflation is greater than 1. That larger than-one coefficient is the mathematical manifestation of explosive inflation.

Chapter 18

1. Suppose the economy has a Phillips curve
$$u = u^n - \alpha(\pi - E\pi).$$

As usual, this implies that if inflation is lower than expected, then unemployment rises above its natural rate, and there is a recession. Similarly, if inflation is higher than expected, then unemployment falls below its natural rate, and there is a boom. Also, suppose that the Democratic party always follows a policy of high money growth and high inflation (call it π_D), whereas the Republican party always follows a policy of low money growth and low inflation (call it π_R).

a. The pattern of the political business cycle we observe depends on the inflation rate people expect at the beginning of each term. If expectations are perfectly rational and contracts can be adjusted immediately when a new party comes into power, then there will be no political business cycle pattern to unemployment. For example, if the Democrats win the coin flip, people immediately expect high inflation. Because $\pi = \pi_D = E\pi$, the Democrats' monetary policy will have no effect on the real economy. We do observe a political business cycle pattern to inflation, in which Democrats have high inflation and Republicans have low inflation.

Now suppose that contracts are long enough that nominal wages and prices cannot be adjusted immediately. Before the result of the coin flip is known, there is a 50-percent chance that

inflation will be high and a 50-percent chance that inflation will be low. Thus, at the beginning of each term, if people's expectations are rational, they expect an inflation rate of $E\pi = 0.5\pi_D + 0.5\pi_R$. If Democrats win the coin toss, then $\pi > E\pi$ initially, and unemployment falls below its natural rate. Hence, there is a boom at the beginning of Democratic terms. Over time, inflation rises to π_D , and unemployment returns to its natural rate. If Republicans win, then inflation is lower than expected, and unemployment rises above its natural rate. Hence, there is a recession at the beginning of Republican terms. Over time, inflation falls to π_R , and unemployment returns to its natural rate.

b. If the two parties take turns, then there will be no political business cycle to unemployment, since everyone knows which party will be in office, so everyone knows whether inflation will be high or low. Even long-lasting contracts will take the actual inflation rate into account, since all future inflation rates are known with certainty. Inflation will alternate between a high level and a low level, depending on which party is in power.

c. The advantage of having an independent central bank set monetary policy is that this bypasses the political business cycle. If elected officials are responsible for setting monetary policy, then they can in theory use policy to their advantage and randomly or arbitrarily set policy in order to help themselves get reelected. If the two parties in this example agree to take turns setting policy, then the result will be some of the same advantages of having monetary policy set by an independent central bank. By taking turns, people know exactly what to expect and so can plan and act accordingly. Inflation will still alternate between a high level and a low level, but it would be predictable and therefore less costly. An independent central bank would have the added advantage of being able at least in theory to always maintain a low and stable inflation rate.

2. There is a time-inconsistency problem with an announcement that new buildings will be exempt from rent-control laws. Before new housing is built, a city has an incentive to promise this exemption: landlords then expect to receive high rents from the new housing they provide. Once the new housing has been built, however, a city has an incentive to renege on its promise not to extend rent control. That way, many tenants gain while a few landlords lose. The problem is that builders might expect the city to renege on its promise; as a result, they may not build new buildings.

Appendix question

a. In the model so far, nothing happens to the inflation rate when the natural rate of unemployment changes.

b. The new loss function is $L(u, \pi) = u^2 + \gamma\pi^2$.

The first step is to solve for the Fed's choice of inflation, for any given inflationary expectations. Substituting the Phillips curve into the loss function, we find:

$$L(u, \pi) = [u_n - \alpha(\pi - E\pi)]^2 + \gamma\pi^2.$$

We now differentiate with respect to inflation π , and set this first-order condition equal to zero: $dL/d\pi = 2\alpha^2(\pi - E\pi) - 2\alpha u^n + 2\gamma\pi = 0$ or,

$$\pi = (\alpha^2 E\pi + \alpha u^n) / (\alpha^2 + \gamma).$$

Of course, rational agents understand that the Fed will choose this level of inflation.

Expected inflation equals actual inflation, so the above equation simplifies to: $\pi = \alpha u^n / \gamma$.

c. When the natural rate of unemployment rises, the inflation rate also rises. Why?

The Fed's dislike for a marginal increase in unemployment now rises as unemployment rises. Hence, private agents know that the Fed has a greater incentive to inflate when the natural rate is higher. Hence, the equilibrium inflation rate also rises.

d. Appointing a conservative central banker means that γ rises. Hence, the equilibrium inflation rate falls. What happens to unemployment depends on how quickly inflationary expectations adjust. If they adjust immediately, then there is no change in unemployment, which remains at the natural rate. If expectations adjust slowly, however, then, from the Phillips curve, the fall in inflation causes unemployment to rise above the natural rate.