MIDTERM #1
APPLIED CALCULUS 120
WINTER 2063

1) If \( f(x) = x^2 + 2x \), find \( f'(1) \) using the limit definition (not derivative rules).

2) Let \( T(t) \) = temperature of water in a pot (in degrees Fahrenheit) \( t \) minutes after the burner is turned on under it. In a paragraph, explain what \( T'(5) \) means. Include explanations of the quantities \( t - 5, T(t) - T(5) \), and \( \frac{T(t) - T(5)}{t - 5} \).

3) Suppose the cost for a factory to produce \( n \) Spiderman action figures is

\[
C(n) = 200,000 + 2n + 4\sqrt{n}
\]

dollars.

a) Find the derivative, \( C'(n) \).

b) \( C'(n) \) is often used to approximate the cost to produce each additional unit if \( n \) are already being produced. This is called the marginal cost of production at level \( n \). What is the marginal cost of production if 4 Spiderman action figures are already being produced? If 4,000,000 are being produced?

c) Using your answers in b), estimate the cost to produce 10 action figures (use marginal cost at level 4). Then estimate the cost to produce 4,000,006 action figures (use marginal cost at level 4,000,000).
3) d) Now calculate the actual costs $C(10)$ and $C(4,000,006)$.

e) Is $C'(n)$ always a good approximation of marginal cost? Explain.

10.4) Find the equation of the tangent line to the graph of the function $f(x) = \frac{2}{x^2}$ at the point $(1,2)$.

10.5) a) If the graph of a cost function looks like:

What is probably true about costs?
or, give a situation where the cost function might look like this.

b) If the graph of a cost function looks like:

What is probably true about costs?
or, give a situation where the cost function might look like this.
6) Find the following derivatives (8 points each)

a) \( \frac{dy}{dx} \bigg|_{x=5} \) if \( y = x^4 - 3x^3 + 7x - 2 \)

b) \( f'(x) \) if \( f(x) = \frac{3}{30x^2 - 10x} \)

c) \( f''(x) \) if \( f(x) = 3x - \frac{2}{x} \)

d) \( f'(7) \) if \( f(x) = (x+1)^{\frac{1}{3}} \)

e) \( \frac{d}{dV} \left( \frac{nRT}{V} \right) \)