I. Derivatives: Find the following derivatives (10 points each)

a) \( \frac{d}{dx}[2^2 \sqrt{5x+7}] \)

b) \( \frac{d}{dx}[5 e^x] \)

c) \( \frac{d}{dx}\left[\frac{2e^2-1}{x^2+1}\right] \)

II. Composition: (5 points each)

a) State the chain rule

b) If \( f(x) = \sqrt{x^2} + 1 \) and \( g(x) = 2x^2 + 1 \), what is \( f(g(x)) \)?

III. Exponents: (5 points each)

a) Simplify the expression \( \left( \frac{5x^2y}{x^3} \right)^2 \)

b) Solve \( 25 = e^{x+2} \) for \( x \).

c) If \( f'(x) = 15f(x) \) and \( f(0) = 3 \), find \( f(x) \).

d) Draw the graph of \( f(x) = 2e^x \) labelling all intercepts.

e) Draw the graph of \( g(x) = e^{-x} \) on the same set of axes, labelling all intercepts.

f) Which graph could represent population? Which could represent radioactivity?
20 points

The following graph represents the growth of a population of bacteria in a test tube.

![Graph](image)

a) What was the population on the 5th day?

b) Estimate how fast the population was increasing on the 3rd day.

c) Indicate on the graph where the population was increasing and decreasing. Where faster? Faster, where slower & slower.

d) What was the largest population and when was it reached? When was it increasing fastest? Decreasing fastest?

e) What days are the 3-day weekend when the grow student forgot to add food to the tube? Explain what might be happening around day 3.
Optimization

A) A hotel in Las Vegas finds that if it charges $80/night for a room, it can rent 300 rooms. If it charges $100/night, it rents only 270 rooms.

a) Assuming a linear demand curve, find price as a function of # rooms to rent.

b) What rate will maximize revenues?

c) If it costs $10 to service a rented room, what rate will maximize profits?

B) Students from Ripon College are to be put into two holding pens, a square one and a circular one. (Circumference of a circle = 2πr, radius, area = πr²)

If 100 feet of fencing are available for this, maximize their discomfort by minimizing the combined area of the pens.

VI Essay (10 points): Write a paragraph answer to one of the following questions

1) Describe an application of the differential equation \( y' = ky \).

2) Explain how cost, revenue, demand, production level and profit are related. Draw graphs of the relevant functions and explain their general properties.