Calculus 120
MIDTERM #2 Practice Exam
SOLUTIONS

I a) \[ \frac{d}{dx} \left[ x^2 \sqrt{5x+7} \right] \]
\[ = \frac{d}{dx} \left[ x^2 \right] \sqrt{5x+7} + x^2 \frac{d}{dx} \left[ \sqrt{5x+7} \right] \]
\[ = 2x \sqrt{5x+7} + x^2 \cdot \frac{1}{2} (5x+7)^{-\frac{1}{2}} \frac{d}{dx} \left[ 5x+7 \right] \]
\[ = 2x \sqrt{5x+7} + \frac{x^2}{2 \sqrt{5x+7}} \cdot 5 \]

b) \[ \frac{d}{dx} \left[ 5e^{\frac{1}{x}} \right] = 5 \frac{d}{dx} \left[ e^{\frac{1}{x}} \right] = 5 \cdot e^{\frac{1}{x}} \frac{d}{dx} \left[ \frac{1}{x} \right] \]
\[ = 5 e^{\frac{1}{x}} \cdot \frac{d}{dx} \left[ x^{-1} \right] = 5 e^{\frac{1}{x}} (-1)x^{-2} = \frac{-5e^{\frac{1}{x}}}{x^2} \]

c) \[ \frac{d}{dx} \left[ \frac{x^2-1}{x^2+1} \right] = \frac{d}{dx} \left[ x^2-1 \right] (x^2+1) - (x^2-1) \frac{d}{dx} \left[ x^2+1 \right] \]
\[ = \frac{2x(x^2+1)-(x^2-1)(2x)}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} \]
\[ = \frac{4x}{(x^2+1)^2} \]
II  a) Look this up - it is in the textbook
   b) \( f(g(x)) = \sqrt{g(x)} - 1 = \sqrt{2x^2 + 1} - 1 \)

III  a) \( \left( \frac{5x^2y}{\sqrt{x}} \right)^2 = \frac{(5x^2y)^2}{(\sqrt{x})^2} = \frac{5^2(x)^2y^2}{x} = \frac{25x^4y^2}{x} \)

   = \( 25x^3y^2 \)

b) \( 25 = e^{x+2} \)
   \( \ln(25) = x+2 \)
   \( \ln(25) - 2 = x \)

c) \( f'(x) = 15f(x) \) means
   \( f(x) = ae^{15x} \)
   \( f(0) = 3 = ae^{15 \cdot 0} = ae^0 = a \cdot 1 \)
   \( = a \)
   So \( f(x) = 3e^{15x} \)

d)

e) \( 2e^x \) could represent population.
   \( e^x \) could represent radioactive.

IV see Winter 2001 midterm #1 solutions
A) a) Two points are \( p(80) = 300 \)  
\[ p(100) = 270 \]
So \( p(x) \) has slope  
\[ \frac{270 - 300}{100 - 80} = \frac{-30}{20} = -\frac{3}{2} \]
So using point-slope,  
\[ y - 270 = -\frac{3}{2} (x - 100) \]
\[ y = -\frac{3}{2}x + 150 + 270 \]
\[ y = -\frac{3}{2}x + 420 \]
\[ p(x) = y = -\frac{3}{2}x + 420 \] is the demand function.

b) Revenues = number \cdot price = x \cdot p(x)  
\[ R(x) = x\left(-\frac{3}{2}x + 420\right) = -\frac{3}{2}x^2 + 420x \]

Maximize this!  
\[ R'(x) = -3x + 420 = 0 \]
\[ x = 140 \text{ rooms} \]
\[ p(140) = -\frac{3}{2} \cdot 140 + 420 = -210 + 420 = $180/night \]

Revenue

The maximum occurs when the hotel charges $180/night.

(We know this is a max, because \( p(x) \)'s graph is a downward opening parabola.)

c) Profit = Revenues - Costs  
\[ P(x) = -\frac{3}{2}x^2 + 420x - 10x = -\frac{3}{2}x^2 + 410x \]
\[ P'(x) = -3x + 410 = 0 \]  
\[ x = \frac{410}{3}, \quad p(\frac{410}{3}) = 215 \]

The maximum profit occurs when the hotel charges $215/night.
B) see Midterm exam #1, 2001 solutions for solution

II write these out on your own. If you would like comments, bring your answers to me.