1) (5 pts each) Solve for $y$ in the following:
   a) $\frac{1}{3} \ln(y^2 + 2y) - \frac{1}{3} \ln(y) = t^2 + c$
   b) $e^{4y^2} - 1 = 3b$

2) (5 pts) Differentiate the expression
   \[ \ln \left( \frac{\sin^2 t \cdot \cos t}{\sqrt{t^2 + 1}} \right) \]

3) (5 pts each) Solve the integrals
   a) $\int_0^{2t} te^t \, dt$
   b) $\int \frac{\cos t}{\sin^2 t} \, dt$

4) (10 pts) Show that the function
   \[ f(t) = \frac{4}{5} \cos t + \frac{8}{5} \sin t + Ce^{-\frac{4}{5}} \]
   satisfies the equation $y' + \frac{y}{2} = 2\cos t$ for all $C$. What is $C$ if $f(0) = 1$?

5) (10 points) Solve the IVP
   \[ \begin{cases} y' = \frac{1}{y} e^t \\ y(0) = 1 \end{cases} \]
6) (5 pts) A skydiver jumps from a plane and opens her parachute. One model for her velocity \( v(t) \) \( t \) seconds after she opens her chute is
\[
m v' = 9.8m - kv
\]
where \( m \) = mass of skydiver, \( 9.8 \) = acceleration from gravity, \( k \) = drag force.

Find the steady state of this equation, determine its stability, and interpret this in terms of the problem.

7) (10 pts) Draw the level curves at \( z = -1, 0, 1, 2 \) for the function \( f(x,y) = y^3 - 3x \).
If this function represents temperatures this morning \( x \) miles north; \( y \) miles east of Baggs Hall, what do the level curves represent?

8) (5 pts each) a) Explain using a diagram what is meant by the least-squares fit line to a set of data.

b) Find the least-squares fit line to the data points
\((-1, 14), (1, 12), (3, 8), (5, 6), (7, 5)\)

using \( m = \frac{n \Sigma xiyi - \Sigma xi \Sigma yi}{n \Sigma x^2 - (\Sigma xi)^2} \) \( b = \frac{\Sigma yi - m \Sigma xi}{n} \).

Graph the data & this line.
9) (5 pts each)
a) Find all partial derivatives of \( f(x, y) = e^{x^2 + y} \)

b) What do \( \frac{\partial f}{\partial x}(1, 2) \) and \( \frac{\partial f}{\partial y}(1, 2) \) mean in terms of the graph of \( f \)\

10) (10 pts) The Cobb-Douglas production function for a new product is given by
\[ f(x, y) = 16x^{0.25}y^{0.75}, \]
where \( x \) = units of labor and \( y \) = units of capital used.

If labor costs \( $50/\text{unit} \) and capital costs \( $100/\text{unit} \), and if \( $500,000 \) has been allocated to production, how should these funds be used to maximize production, and what is the largest number of units which can be produced?

11) (10pts) Find the three critical points for
\[ f(x, y) = x^3 - 3xy^2 + 6y^2 \]
and determine if each is a maximum, minimum, or neither.

(Hint: solve \( \frac{\partial f}{\partial y} = 0 \) first - there are two possible cases - then use the equations in those two cases to solve \( \frac{\partial f}{\partial x} = 0 \). You will find two critical points in one case and one critical point in the other case.)