Selected HW solutions
HW # 7

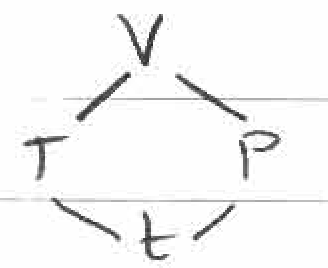
14.5 41) $PV = nRT$ is the ideal gas equation

$n = 1$ mole here $\quad R \approx 8.31$

$\frac{dP}{dt} = .05$ kPa $\quad \frac{dT}{dt} = .15$ K/s

Find $\frac{dV}{dt}$ when $P = 20$ kPa, $T = 320$K

$$V = \frac{nRT}{P} = \frac{8.31T}{P}$$



$$\frac{dV}{dt} = \frac{\partial V}{\partial T}\frac{dT}{dt} + \frac{\partial V}{\partial P}\frac{dP}{dt}$$

$$= \frac{8.31}{P} \cdot .15 + \frac{-8.31T}{P^2} .05$$

$$= \frac{8.31}{20} \cdot .15 + \frac{-8.31(320)}{(20)^2} .05$$

$$\approx -.27$$

The volume is decreasing at ~ .27 litres/s

14.6 33) $V(x,y,z) = 5x^2 - 3xy + xyz$

Find $D_{\vec{u}}V(3,4,5)$ where $\vec{u}$ is the unit vector in the direction $i + j - k$

$$u = \frac{\langle 1,1,-1\rangle}{\sqrt{3}} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle$$

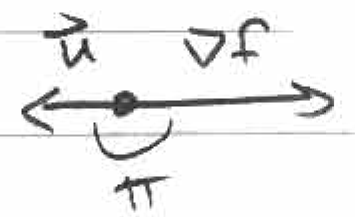
$$\nabla V = \langle 10x - 3y + yz, -3x + xz, xy \rangle$$

$$\nabla V(3,4,5) = \langle 30-12+20, -9+15, 12 \rangle$$
$$= \langle -2, 6, 12 \rangle$$

$$D_{\vec{u}} V(3,4,5) = \nabla V(3,4,5) \cdot \vec{u}$$
$$= \langle -2, 6, 12 \rangle \cdot \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle$$
$$= \frac{-2}{\sqrt{3}} + \frac{6}{\sqrt{3}} + \frac{-12}{\sqrt{3}} = \frac{-8}{\sqrt{3}}$$

27) a) We are looking for the direction $\vec{u}$ for which $D_{\vec{u}} f$ is minimized.

$D_{\vec{u}} f = \nabla f \cdot \vec{u} = |\nabla f| \cdot |\vec{u}| \cdot \cos\theta$ which is minimized when $\cos\theta$ is minimized, ie, when $\theta = \pi$, $\cos\theta = -1$. If $\theta = -1 =$ angle between ∇f and $\vec{u}$, this means



They point in opposite directions. But $-\nabla f$ also points in the opposite direction as ∇f, so $-\nabla f$ points in the direction of $\vec{u}$, which is the direction in which $D_u f$ is minimized, ie, the direction of steepest descent.

b) $\nabla f = \langle 4x^3y - 2xy^3, x^4 - 3x^2y^2 \rangle$

$-\nabla f(2,-3) = -\langle 4 \cdot 8 \cdot -3 - 2 \cdot 2 \cdot -27, 16 - 3 \cdot 4 \cdot 9 \rangle$

$= \langle -12, 92 \rangle$ is the direction of steepest descent.