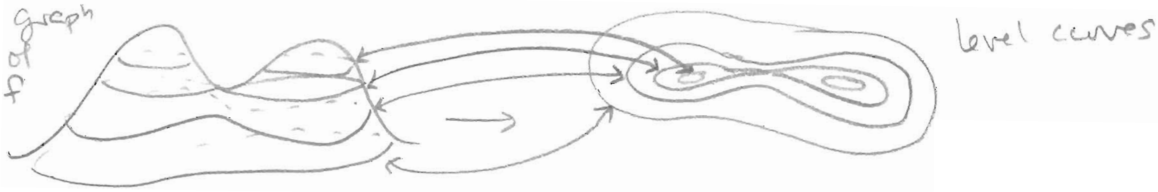
Quiz #5
Calculus 160
Spring 2006, Hunsicker

Name KEY IHRTLUHC

1) Define level curves of a function of two variables and explain what the definition means using diagrams.

The Level curves of a function $f(x,y)$ are the curves in the x-y plane with equations $f(x,y)=k$ for $k \in \mathbb{R}$.

We can use level curves to give a 2 dim'l representation of the graph of f:



2) Find the equation of the tangent plane to the graph of $f(x,y) = e^{xy} - \sin(y) + 2x^2 y$ at the point (5, 0, 1). Then use it to approximate the value of f(x,y) when x=4.9 and y=0.2.

The equation is $z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

where $(x_0,y_0,z_0) = (5,0,1)$

and $f_x(x,y) = ye^{xy} - 4xy$ $\qquad$ $f_y(x,y) = xe^{xy} - \cos y + 2x^2$

$f_x(5,0) = 0$ $\qquad$ $f_y(5,0) = 5e^{0.5} - \cos 0 + 2\cdot(5^2)$

$= 54$

So $z - 1 = 0(x-5) + 54(y-0)$

$z = 54y$ is the equation of the tangent plane.

at $x=4.9$, $y=.2$,

$f(4.9, 0.2) \approx z = 54 \cdot .2 = \frac{54}{5}$.

Quiz #6
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1) Prove that the gradient of f(x,y) is perpendicular to the level curves of f(x,y).

Parametrize a level curve $f(x,y)=k$ by $\vec{r}(t)=\langle x(t),y(t)\rangle$.
Then since f is constant along this curve,
$f(x(t),y(t))=k$ for all t. Take the derivative with respect to both sides:

$$\frac{d}{dt}\left(f(x(t),y(t))\right)=\frac{d}{dt}(k)=0$$
$$\parallel$$
$$f_x(x(t),y(t))\,x'(t)+f_y(x(t),y(t))\,y'(t)=\langle f_x(x,y),f_y(x,y)\rangle\cdot\langle x'(t),y'(t)\rangle$$
$$=\nabla f(x,y)\cdot\text{Tangent to curve}$$

So all together, $\nabla f\cdot\text{Tangent}=0$, ie $\nabla f\perp$ tangent, which means $\nabla f\perp$ level curves

2) Find all local maxima and minima of the function $f(x,y)=4xy^2-x^2y^2-xy^3$ using the first and second derivative tests for functions of two variables.

$$\nabla f(x,y)=\langle 4y^2-2xy^2-y^3,\ 8xy-2x^2y-3xy^2\rangle=\langle 0,0\rangle$$

$$\Rightarrow\begin{cases}4y^2-2xy^2-y^3=0\\ 8xy-2x^2y-3xy^2=0\end{cases}\Rightarrow\begin{cases}y^2(4-2x-y)=0\\ xy(8-2x-3y)=0\end{cases}\Rightarrow\begin{cases}y=0 \text{ or } 4-2x-y=0\\ x=0,\ y=0,\text{ or } 8-2x-3y=0\end{cases}$$

so we have stationary points $(x,0)$ for any x, (if $y=0$)
or $(0,4)$ (if $4-2x-y=0$ and $x=0$) or $(1,2)$ (if $4-2x-y=0=8-2x-3y$)

$f_{xx}=-2y^2$, $f_{xy}=8y-4xy-3y^2$, $f_{yy}=8x-2x^2-6xy$, so

$Df(x,y)=f_{xx}f_{yy}-(f_{xy})^2=(-2y)(8x-2x^2-6xy)-(8y-4xy-3y^2)^2$
$Df(x,0)=0$, so we can't tell about these (sorry - don't worry about them)
$D(0,4)=-(8\cdot4-3\cdot4^2)^2<0 \Rightarrow$ this is a saddle
$D(1,2)=(-4)(8-2-12)-(16-8-12)^2=48-16>0$, $f_{xx}=-2\cdot2=-4<0$
so $(1,2)$ is a local max.