Calc 160 Spring 2005

Hw#1 selected solutions

12.1 19) a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is $m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

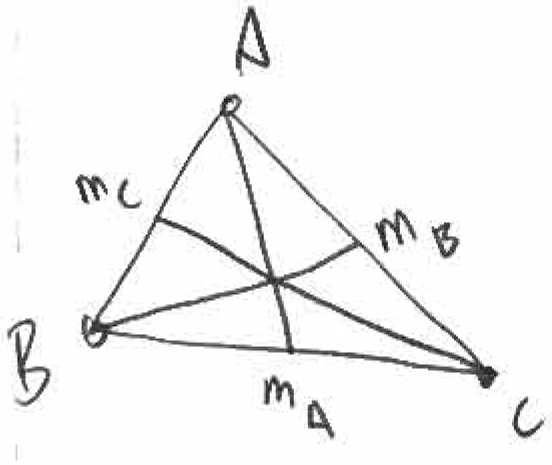
There are several ways to do this problem. We will do it by showing that $\overrightarrow{P_1m} = \frac{1}{2}\overrightarrow{P_1P_2}$. This implies the vectors have the same direction, ie, the points P_1, m, P_2 are colinear, and that the distance from P_1 to m is $\frac{1}{2}$ the distance from P_1 to P_2.

$$\overrightarrow{P_1m} = \left\langle \left(\frac{x_1+x_2}{2}\right) - x_1, \left(\frac{y_1+y_2}{2}\right) - y_1, \left(\frac{z_1+z_2}{2}\right) - z_1 \right\rangle$$

$$= \left\langle \frac{x_2-x_1}{2}, \frac{y_2-y_1}{2}, \frac{z_2-z_1}{2} \right\rangle = \frac{1}{2}\langle x_2-x_1, y_2-y_1, z_2-z_1\rangle$$

$$= \frac{1}{2}\overrightarrow{P_1P_2}.$$

b) Find the lengths of the medians of the triangle with vertices $A(1,2,3)$, $B(-2,0,5)$ and $C(4,1,5)$



medians are $\overline{m_C C}$, $\overline{m_B B}$ and $\overline{m_A A}$.

$$m_A = \text{midpoint of } BC = \left(\frac{-2+4}{2}, \frac{0+1}{2}, \frac{5+5}{2}\right) = \left(1, \frac{1}{2}, 5\right)$$

$$m_B = \text{midpt of } AC = \left(\frac{1+4}{2}, \frac{2+1}{2}, \frac{3+5}{2}\right) = \left(\frac{5}{2}, \frac{3}{2}, 4\right)$$

$$m_C = \text{midpt of } AB = \left(\frac{1+-2}{2}, \frac{2+0}{2}, \frac{3+5}{2}\right) = \left(-\frac{1}{2}, 1, 4\right)$$

lengths of medians are:

$|m_A A| = \text{dist}\left((1, \tfrac{1}{2}, 5), (1,2,3) \right)$

$= \sqrt{(1-1)^2 + (\tfrac{1}{2}-2)^2 + (5-3)^2} = \sqrt{0 + \tfrac{9}{4} + 4}$

$= \sqrt{\tfrac{25}{4}} = \tfrac{5}{2}$

$|m_B B| = \text{dist}\left((\tfrac{5}{2}, \tfrac{3}{2}, 4), (-2, 0, 5) \right)$

$= \sqrt{(\tfrac{5}{2}-(-2))^2 + (\tfrac{3}{2}-0)^2 + (4-5)^2}$

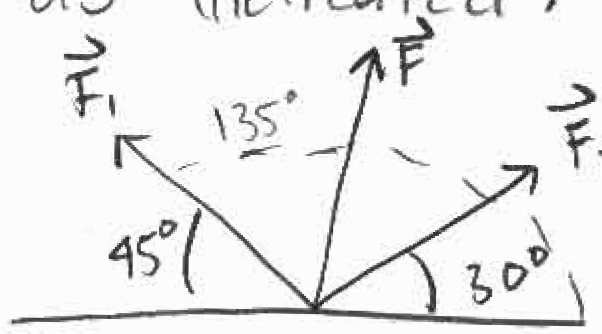
$= \sqrt{\tfrac{81}{4} + \tfrac{9}{4} + 1} = \sqrt{\tfrac{94}{4}} = \tfrac{\sqrt{94}}{2}$

$|m_c C| = \text{dist}\left((-\tfrac{1}{2}, 1, 4), (4, 1, 5) \right)$

$= \sqrt{(-\tfrac{1}{2}-4)^2 + (1-1)^2 + (4-5)^2}$

$= \sqrt{\tfrac{81}{4} + 0 + 1} = \sqrt{\tfrac{85}{4}} = \tfrac{\sqrt{85}}{2}$

29) F_1 has magnitude 10lb, F_2 has magnitude 12 lb as indicated:



Find magnitude & direction of resultant force, $\vec{F}$.

$\vec{F}_2 = \langle 12\cos 30°, 12\sin 30° \rangle = \langle 6\sqrt{3}, 6 \rangle$

$+\ \vec{F}_1 = \langle 10\cos 135°, 10\sin 135° \rangle = \langle -5\sqrt{2}, 5\sqrt{2} \rangle$

$\vec{F} = \langle 6\sqrt{3} - 5\sqrt{2}, 6 + 5\sqrt{2} \rangle = (6\sqrt{3}-5\sqrt{2})\vec{i} + (6+5\sqrt{2})\vec{j}$

29 cont'd)

So $|F|$ = strength of resultant force

$$= \sqrt{(6\sqrt{3} - 5\sqrt{2})^2 + (6 + 5\sqrt{2})^2}$$

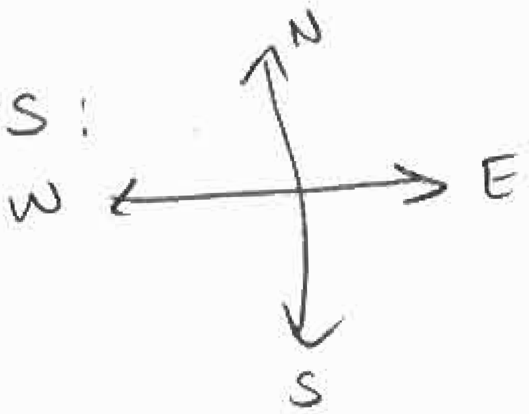
$$\approx 13.5$$

and

$$\tan\theta = \frac{y}{x} = \frac{6+5\sqrt{2}}{6\sqrt{3}-5\sqrt{2}}$$

so $\theta = \arctan\left(\frac{6+5\sqrt{2}}{6\sqrt{3}-5\sqrt{2}}\right) \approx 76^\circ$

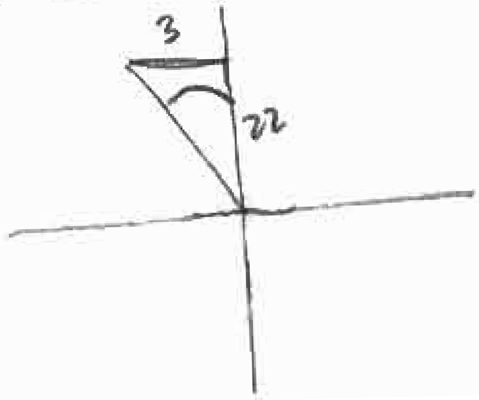
31) Assuming axes:



The velocity vector of the boat relative to the water is $\langle 0,22\rangle$. The velocity vector of the woman relative to the boat is $\langle -3,0\rangle$. So the velocity of the woman relative to the water is $\langle 0,22\rangle + \langle -3,0\rangle = \langle -3,22\rangle$.

Her speed is $|\text{velocity}| = \sqrt{(-3)^2 + (22)^2} \approx 22.2$ mi/hr

Her direction is N θ° W where



$\tan\theta = \frac{3}{22}$ so $\theta \approx 8$

so her direction is N 8°W