

Cycles in the Simplex Algorithm

The *Simplex Algorithm* solves problems that are called *linear programs* – problems in which we want to minimize or maximize a linear function subject to linear inequalities and/or equations. You can find many, many references to such problems by a simple search. Under normal circumstances, the simplex algorithm moves fairly quickly to a solution. However, it is possible for the algorithm to cycle endlessly, without making progress. I am studying how that can happen. This is a subject that many others have studied before; I am working on a computer search algorithm for general systems of inequalities that can lead to cycling.

Without getting more into the technical details, I will describe the image: it stands for a sequence of matrices all representing the coefficient matrix of the same system of three linear equations in five variables; each equation has 0 right side, and so the right sides are not depicted. Each 3×5 matrix in sequence is obtained from the previous one by identifying a pivot and clearing above and below that pivot – exactly as is done in the Elimination Algorithm. The plus signs stand for coefficients that are known to be positive, the minus signs for those known to be negative, the zeros for those known to be zero, and the k for unknown! This pattern was obtained by a computer search for possible cycles in the simplex algorithm. I am working on fine-tuning the computer search so that *possible cycles* can be tested to see if they are actual cycles.