U039
SEMANTICS, POSSIBLE-WORLDS

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NOTES:

1. Logical symbols throughout.
2. Not sure how one is supposed to to point to required background (see note just after B-section).
3. This piece, along with SITUATION SEMANTICS, has been separated out from U039, SEMANTICS, MODEL-THEORETIC (6K words). Separated entries was the original idea, so I assume it will be ok to revert to that.

Word count: approximately 3800 not including references.

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Possible worlds semantics (PWS) is a family of methods that have been used to analyze a wide variety of *intensional* phenomena, including modality, conditionals, tense and temporal adverbs, obligation, and reports of informational and cognitive content. PWS spurred the development of philosophical logic and led to new applications of logic in computer science and artificial intelligence. It revolutionized the study of the semantics of natural languages. PWS has inspired analyses of many concepts of philosophical importance, and the concept of a possible world has been at the heart of important philosophical systems. (See also POSSIBLE WORLDS, PHILOSOPHICAL ISSUES IN MODAL LOGIC, INTENSIONALITY, INTENSIONAL LOGIC, MODEL THEORY.)

This entry assumes some familiarity with symbolic logic (see SYMBOLIC LOGIC).

1 Intensions Demeaned

Traditionally, the *intension* of a predicate was distinguished from its *extension*; the former is a property, the latter is a set. The predicates “is a featherless biped that is not a plucked chicken” and “is human” have (one can imagine) the same extensions but different intensions. (The example is Bertrand Russell’s.) Gottlob Frege’s concepts of *sinn* and *bedeutung* extend this idea: the sinn of a singular term is an identifying condition (or ‘individual concept’), the bedeutung the individual designated. The sinn of a sentence is a proposition, the bedeutung a truth value. Frege defended his choice of truth-values as the bedeutung of sentences on systematic grounds. (See FREGE.)

As model theory was developed by Tarski and others, a version of Frege’s choices for *bedeutung* became the standard values in *extensional semantics*. The extension of an n-place predicate is the set of n-tuples of objects of which the predicate is true (thus, the extension of ‘gives’ might be the set of those four-tuples containing two persons, an object, and a time, such that the first person gives the object to the second person at that time). The extension of a singular (object-denoting) term is the object it designates. The extension of a sentence is a truth-value. The package of the predicate calculus with an extensional semantics proved adequate for important work in mathematical logic and crowded older approaches to logic out of the classroom. In contrast, no generally agreed upon understanding of intensions emerged. In the middle part of the century interest in intensional phenomena waned. In fact, the success of extensional logic led to somewhat uncharitable attitudes towards any non-extensional phenomena. On Quine's influential view non-extensional constructions are not suited for scientific work; they are more in need regimentation rather than straightforward analysis. (Non-extensional constructions are those that apparently distinguish among fillings that have the same extensions. One example is "Elwood believes that ...". "Elwood believes that Stanford is east of Hawaii" might be true, while "Elwood believes that Stanford is east of Berkeley" might be false, even though "Stanford is east of Hawaii" and "Stanford is east of Berkeley" both have the same extension (the truth-value true). "Intensional" is sometimes used simply to mean "non-extensional," and is sometimes given a narrower meaning.)

2 Modal Logic

A number of philosophers and logicians continued to attempt to provide straightforward analyses of intensional phenomena, however. Until the 1950's, the emphasis was on syntactic approaches. A key figure was C.I. Lewis, whose dissatisfaction with the extensional treatment of "if ... then ..." as the material conditional led him first to the logic of "strict implication", then to *modal logic*, the logic of necessity and possibility (Lewis and Langford). The modal operators (typically translated 'Necessarily' and 'Possibly'), usually symbolized as $\Box$ and $\Diamond$, are not truth-functional, and so require intensional analysis.

The language of propositional modal logic (ML) consists of the language of propositional logic, plus the rule that if ϕ is a well-formed formula (wff), then so are $\Box\phi$ and $\Diamond\phi$. Lewis and others worked out a number of axiom systems for ML and studied and compared them proof-theoretically.

More semantically oriented approaches to intensionality emerged later in the century, beginning with Carnap (1946 and 1947). One of the most important of Carnap's many contributions to the study of intensionality was to recruit Leibniz's idea that necessary truth was truth "in all possible worlds" to the task of building an intensional semantics. This is the guiding idea of possible worlds semantics. Carnap's version of this idea, less straightforward than those that were to follow, relies on linguistic representations of possible worlds which he called "state-descriptions".

The basics of the now-standard treatment came in the late 1950's and early 1960's with results obtained by Stig Kanger (1957; see also Follesdal 1994), and Saul Kripke (1959, 1963a, 1963b, 1965). (See also Hintikka 1957, Montague 1974b.)

We'll look briefly at **K**, **S4**, and **S5**, three among the plethora of axiom systems for modal logic that have been studied.

S5 includes:

- all propositional tautologies and modus ponens,
- the definition: $\Diamond\phi =_{df} \neg\Box\neg\phi$,
- the rule of Necessitation: ($\vdash \phi / \vdash \Box\phi$; i.e., if ϕ is deducible from the null set of premises, then so is $\Box\phi$),
- the axioms:

$$K : \Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$$

$$T : \Box\phi \to \phi$$

$$\mathit{4} : \Box\phi \to \Box\Box\phi$$

$$B : \Diamond\Box\phi \to \phi$$

If we drop B we have **S4**; if in addition we drop $\mathit{4}$ and T we have **K**, Kripke's minimal system.

A modal model structure is a pair $\langle K, R\rangle$. K is the set of worlds; R we will get to in a minute. A modal model will tell us which atomic sentences of the base language L are true at which worlds of K. For the connectives of propositional logic the rules remain unchanged. To extend the system to include $\Box$ it is natural, on the Leibnizian conception, to use the rule (we symbolize 'ϕ is true in w' as $\phi[w]$):

$\Box_A$: $\Box\phi[w]$ iff $\forall w'\phi[w']$.

Given the definition of $\Diamond$ we have

$\Diamond\phi[w]$ iff $\exists w'\phi[w']$.

The reader can check that on this conception, all of the axioms for S5 are valid, i.e. true in every world of every model. Consider B, for example. Suppose the antecedent is true at w. Then $\exists w'\forall w''\phi[w'']$. Since the existential quantification is vaccuous, this reduces to $\forall w''\phi[w'']$. Then, by U.I., we have $\phi[w]$.

Note that the $\Diamond$ was vaccuous; $\Diamond\Box\phi$ collapsed into $\Box\phi$. This is characteristic of **S5**: iterated modalities collapse to the right.

S5 is a natural logic for metaphysical necessity, which was doubtless the conception Leibniz had in mind. But there are other coherent concepts of necessity, for which some of the axioms of **S5** do not seem correct, and for which distinctions among iterated modalities are significant.

Consider physical necessity. We have $\Box\phi[w]$ if ϕ is true in every world that obeys the laws of physics of w. Suppose w' is a world in which has all of our laws and more. Certain events may be ruled out by the physics of w' that are not ruled out by our physics, so our world is not physically possible relative to w'. Suppose, for example, that it is a law of physics in w' that no golf ball travels more than 200 yards. (The reader may come up with a more plausible example.) Then even though w' is physically possible (it obeys our laws), and "No golf ball travels over 200 yards" is necessary in w', it's not true that no golf ball travels over 200 yards. So Axiom B isn't correct for physical necessity.

T is intuitive in the case of metaphysical and physical necessity, but not for *deontic logic*, in which "$\Box\phi$" in interpreted as "It ought to be the case that ϕ" (see also DEONTIC LOGIC).

In discussing these alternatives conceptions of necessity and possibility, we move from an absolute to a relative conception of possibility, the idea that the set of worlds relevant to issues of necessity varies from world to world. This is the information given by the second member of the model structure above. R is a relation on K, the "accessibility relation". Different accessibility relations correspond to different conceptions of necessity. We replace our absolute rule $\Box_A$ with a relative rule:

$\Box_R$: $\Box\phi[w]$ iff $\forall w'$, if w' is accessible from w, then $\phi[w']$

The axioms that characterize the various systems of modal logic correspond to the logical properties of the relation R. The axiom K places no restrictions on it. T requires reflexivity. *4* requires transitivity, and B symmetry. Thus absolute necessity, captured by **S5**, is the case where the accessibility relation is an equivalence relation. (See also MODEL THEORY, MODAL LOGIC.)

3 Other Applications

The semantical apparatus developed for modal logic has been used to investigate a number of other logical systems.

In *epistemic logic*, for example, a knowledge operator, indexed by knowers, is patterned after $\Box$; $K_\alpha\phi$ means "ϕ holds in all of α's epistemic alternatives" (Hintikka 1962; see also EPISTEMIC LOGIC).

It is important for the philosophically oriented reader to keep in mind that for the purposes of developing and applying semantical treatments of intensional languages, for example, in completeness proofs, the possible worlds of possible worlds semantics need not be invested with any important metaphysical properties; they are just indices for models. The basic apparatus has been used to study a number of areas in which the metaphor of a possible world is inapplicable. In *dynamic logic*, for example, the apparatus of modal logic is applied to programs. The "worlds" are states of a machine. Accessibility relations are indexed by programs. Where α is a program, $\Box[[\alpha]]\phi$ means "ϕ holds after every terminating execution of α" (see Pratt 1976; see also DYNAMIC LOGIC).

The interplay between semantic structures and logical systems involved in these investigations constitute a development in logic comparable to the move in geometry away from Euclidean Geometry, conceived as the one true system, to geometry as the study of alternative axiom systems for spaces with diverse properties.

4 Temporal Logic

The apparatus of modal model structures works nicely to provide a semantics for temporal logic—the logic of operators modeled after the tense and temporal adverb systems of natural languages. Let "G" mean "it will always be the case" and "F" mean "it will sometime be the case"; "F" can be defined as "$\neg G \neg$". Thus G is a universal operator, analogous to $\Box$, and F is an existential operator, analogous to $\Diamond$. Similarly, let "H" mean "it has always been the case" and define "P" as "$\neg H \neg$". Then instead of a set of worlds and an accessibility relation, take a model structure to be a set of moments of time, and an ordering relation between them. The need for an accessibility relation is rather more intuitive here than in the case of necessity and possibility since, unlike worlds, we usually think of times as ordered, by the relation of *before*. As with modal logic, different logics correspond to different conceptions of the ordering relation. One minimal tense logic (van Benthem 1988) contains the axioms:

- $G(\phi \to \psi) \to (G\phi \to G\psi)$
- $H(\phi \to \psi) \to (H\phi \to H\psi)$
- $\phi \to GP\phi$
- $\phi \to HF\phi$

and the rules modus ponens and the analogue to Necessitation, sometimes called "Eternity":

- $\vdash \phi / \vdash G\phi$;
- $\vdash \phi \; / \vdash H\phi$.

As with modal logic, there is a precise correspondence between ordering conditions and additional axioms. For example $PP\phi \leftrightarrow P\phi$, which seems plausible enough, requires that the structure of moments be transitive (if t is before t' and t' is before t'', t is before t'') and dense (if t is before t', there is a t'' between them, i.e., after t and before t'). (See Prior 1959, 1967, 1977, van Bentham 1988; see also TENSE AND TEMPORAL LOGIC.)

5 Conditional Logic

As we noted, dissatisfaction with the material conditional as an explication of the ordinary language conditional was an early complaint against extensional logic. There is some connection between the antecedent and the consequent that the semantics for the material conditional misses. For one thing, $\phi \to \psi$ is true whenever ϕ is false, making all counterfactual conditionals trivially true. Let's use "$\Rightarrow$" as a symbol for a better approximation. Another key way in which $\Rightarrow$ should differ from $\to$ is that it should not alwaysa permit *strengthening the antecedent*. We have

$\phi \to \psi$ only if $(\phi \wedge \chi \to \psi)$

but not:

$\phi \Rightarrow \psi$ only if $(\phi \wedge \chi \Rightarrow \psi)$

For example:

ϕ: I put water in my canteen when I start my hike

ψ: I have water when I stop for a rest

χ: There is a hole in my canteen

Frank Ramsey's test for evaluating conditionals, massaged a bit by Robert Stalnaker, is "make the minimal revision of your stock of beliefs required to assume the antecedent. Then, evaluate the acceptability of the consequent on the basis of this revised body of beliefs" (Ramsey 1931). Stalnaker (1968) and David Lewis (1973) have proposed analyses that implement this idea within the possible worlds framework:

Stalnaker: $\phi \Rightarrow \psi[w]$ iff ψ holds in *the* ϕ-world that is closest to w.

Lewis: $\phi \Rightarrow \psi[w]$ iff ψ holds in *all* ϕ-worlds which are closest to w.

These analyses require a a relation of overall similarity or closeness among worlds. (It can be argued that the relation of overall similarity of worlds is vague and context sensitive; it is replied that this captures the vagueness and context sensitivity of the ordinary conditional.)

On either analysis, strengthening the antecedent fails for $\Rightarrow$, because when the antecedent is strengthened, different worlds might be the closest in which the antecedent is true.

The choice between Lewis's definition and Stalnaker's depends on such issues as whether there is always a unique closest world. One important principle that turns on this is Conditional Excluded Middle,

$$(\phi \Rightarrow \psi) \vee (\phi \Rightarrow \neg\psi)$$

which Stalnaker endorses and Lewis rejects. (See also CONDITIONALS, COUNTERFACTUALS.)

6 Quantified Modal Logic

In 1946, Ruth Barcan Marcus and Rudolf Carnap independently published systems of Quantified modal logic (QML), in which principles like the following were considered:

$$\forall x \Box \phi(x) \rightarrow \Box \forall x \phi(x)$$

This is the "Barcan Formula", attractive to those who would reduce de re to de dicto necessity.

Kripke (1963b) has provided a semantics for these systems. A quantificational modal structure $\langle K, R, \Psi \rangle$ adds a function Ψ which assigns a domain of individuals to each possible world in K. A model assigns extensions to each predicate at each world. On the natural, "world-bound" interpretation of universal quantification, $\forall x \phi(x)$ is true in a world w iff in w $\phi(x)$ is true of every member of the domain of w.

If we suppose, as is natural, that worlds have varying domains, that objects that might have been in our world, for example, actually exist in other worlds, then it seems the Barcan formula is not valid. Even if every object we find in the actual world is ϕ in all possible worlds, there could objects in other possible worlds that are not ϕ. But the formula can be validated, if we interpret quantifiers as ranging over all possible objects, or if we suppose that the domain is constant across worlds, and all of these alternatives have found champions.

"Quantifying in," that is, quantification across modal operators as in the antecedent of the Barcan formula, was deemed by Quine to put us in danger of a commitment to "essentialism".

Suppose Quine is in the extension of "is a Kantian" in some possible world w. This fact about the realm of possibility is not a fact about the necessary or possible truth of some sentence, so there is some commitment to de re modality. Note however that accepting such facts as these does not commit one to a view that Quine must have an essence, that distinguishes him from all other objects, one natural understanding of "essentialism" (Follesdal 1986). (See also PHILOSOPHICAL ISSUES IN MODAL LOGIC.)

7 Index Theory and Intensional Logic

Starting in the late 1960's, PWS began to find a role beyond providing semantics for particular logical systems. One development was the development of *intensional logics* that combined modal, temporal and other operators.

We can think of time and possible worlds, for example, as two dimensions along which the truth of a sentence can vary; from this point of view, it is natural to provide a semantics for a system in which sentences are true in worlds at times, and that contains both temporal and modal operators. Thus, for example,

$$\Box(H\phi)$$

will be true in world w at time t iff in every world w' accessible to w, at every time t' earlier than t, ϕ is true at w' and t'.

But there are also other factors relative to which the truth of sentences can vary. In particular, sentences containing indexicals ("I", "you", "here", "now") will vary in truth depending on who says them, to whom, where and when.

Thus we could think of

I will walk to the store

as being true at a world w, time t and person a, if in w, a walks to the store at some time subsequent to t.

Following the advice of Dana Scott, Montague, Lewis and others developed and explored versions of *index theory*, systems in which sentences were true at an index, where indices were n-tuples of worlds, times, speakers, and other factors (see Scott 1970, Montague 1974c, Lewis 1970).

Kamp's (1971) *double-indexing*, Segerberg's (1973) *two dimensional modal logic* and Kaplan's (1989) *logic of demonstratives* provide alternatives to index theory. These authors emphasize the difference between *reliable* sentences such as "I am here now", that, even if they express contingent propositions, cannot be uttered falsely, and sentences such as 'either there are cats or there are not', that are valid in the standard sense, of being true in every world in every modal model.

8 Montague Semantics

Richard Montague was a leader in the development of intensional logic. In his early papers, he developed PWS as a tool for investigating a number of phenomena of philosophical interest, such as sense-data and events (1974d). In later work he developed PWS as a powerful tool for model-theoretic treatments of the semantics of English and other natural languages. Montague's work has had a profound influence in linguistics (see Partee 1989).

As the body of Montague's work developed, intensional phenomena were increasingly seen not as exceptional and marginal, but as at the core of the way language works. In modal logic, intensionality derives from special operators added onto a base language that works on extensional principles. In Montague semantics, intensionality is a ubiquitous phenomenon; there are not only intensional operators, but intensional verbs, adjectives, adverbs, and so on. In Montague's later work, intensionality is basically the default case, with special postulates to guarantee extensionality (1974e, 1974f).

By using intensional logic and possible worlds semantics to give a precise semantics for constructions of natural language, Montague developed an important new subdiscipline of linguistics, often simply called "Montague Grammar".

9 Intensions Triumphant

Possible worlds semantics provides philosophy with a toolkit of entities for the analysis of intensional phenomena:

- For individual concepts: functions from worlds to individuals.
- For properties: functions from worlds to extensions;
- For propositions: functions from worlds to truth-values (or sets of worlds).

Note that these functions are themselves extensionally understood, and so analyzable within the framework of set theory, and in that sense at least, free of obscurity.

By the 1970's, philosophers were availing themselves of these tools to talk in disciplined ways about many traditional and some new issues, issues often not directly connected with the interpretation of systems of logic. To mention just a few:

- Quantified modal logic has been at the heart of a productive rethinking of issues involved in the distinction between de dicto and de re necessity, beginning with Quine's charge that quantified modal logic commits us to essentialism. (See Follesdal 1986, Plantinga 1974; see also ESSENTIALISM).

- Possible worlds semantics provides two models for the semantics of names. On the possible worlds version of the Frege-Russell-Searle descriptive account of names, the meaning of a name is an individual concept. An alternative is to model names on variables directly assigned to individuals, the same for all worlds, irrespective of their properties in the worlds. Marcus suggested the latter possibility (1961), and in *Naming and Necessity*, Saul Kripke has mounted a full-scale challenge to the descriptive account, arguing that names are "rigid designators" (referring to the same object in all worlds), and providing a causal account of the link between name and thing as an alternative to the descriptive account. (See also PROPER NAMES.)

- David Kaplan, whose lectures, seminars, and unpublished writings stirred much interest in possible worlds semantics throughout the 1970's, worked out an account of indexicals and demonstratives in the context of index theory. This has led to a clarification of a number of issues involving the semantics and epistemology of indexicals. (See Kaplan 1989, Perry 1993a, Stalnaker 1981; see also INDEXICALS AND DEMONSTRATIVES.)

- David Lewis has used the apparatus of possible world semantics to make significant contributions to our understanding of convention, the semantics of natural language, the understanding of counterfactuals (see above), and many other issues in metaphysics, epistemology and the philosophy of science. (See Lewis 1969, 1970.)

  Lewis's own view of possible worlds (1986) maintains that possible worlds are alternative concrete realities; they are actual for their inhabitants, as ours is for us. The inhabitants of other worlds are not identical with the inhabitants of the actual world (i.e., our world), but are their *counterparts*; Lewis's account of quantification is based on the counterpart relations rather than identity. (See also DAVID LEWIS, POSSIBLE WORLDS.)

10 All the Intensions We Need?

Can possible worlds semantics supply philosophy with all the intensions that are needed to understand intensional phenomena? Can all intensions be understood as functions from worlds to appropriate sets?

One of the most difficult challenges is the problem of propositional attitudes. The basic problem is that possible worlds semantics supplies only one necessary proposition, the set of all worlds, and only one contradictory proposition, the null set. This seems to pose a severe problem for dealing with mathematical knowledge. Given usual principles of compositionality, we could infer from "Elwood knows that $7+5=12$" to "Elwood knows that S" for any true mathematical sentence S, or any other necessary truth for that matter.

Robert Stalnaker (1984) has given an careful and extended defense of the use of possible worlds semantics in epistemology. He argues that the concept of content needed for propositional attitudes is grounded in pragmatic relations and such informational relations as indication. The problem of mathematical knowledge, he argues, can be resolved by seeing a linguistic element in our knowledge of mathematical truths.

Advocates of situation semantics have argued that the possible worlds analysis of indication is also vitiated by the problem of the single necessary proposition, however. Where P is a a contingent proposition and N is the necessary proposition, $P = P\&N$. So if the tree rings indicate that the tree is one hundred years old, they also indicate that it is one hundred years old and $7+5=12$. But indication appears to distribute over conjunction, so we could infer that the tree rings indicate that $7+5=12$. (See Perry 1993c, 1993d; see also SITUATION SEMANTICS.)

While these and other problems have stimulated great interest in other approaches to intensionality in recent years, (see PROPERTIES) it seems fair to say that possible worlds semantics has had by far the most impact on the disciplined investigation of intensional phenomena and that no alternative treatment yet devised provides as natural and comfortable a scheme for thinking about matters intensional.

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